Day 25 **IGCSE 10-a-day: Model Solutions**

1 Depreciation

A car is bought for £20,000. Each year its value falls by 30%.
After how many whole years is it first worth less than a quarter of its original value? (Calculator)

Target value = £5,000
After 3 years: £6,860 (not less)
After 4 years: £4,802 (less)
Answer: 4 years

2 Expand then differentiate

$y = x(x - 1)(3x - 3)$
Find $\frac{dy}{dx}$.

Expand: $y = 3x^3 - 6x^2 + 3x$
$3x^3 \to 9x^2$, $-6x^2 \to -12x$, $3x \to 3$
$\frac{dy}{dx} = 9x^2 - 12x + 3$
Answer: $\mathbf{\frac{dy}{dx} = 9x^2 - 12x + 3}$

3 Cyclic quadrilaterals

A, B, C and D are points on a circle. Angle DAB = $(4x - 5)°$, angle BCD = $(5x + 32)°$ and angle ABC = $(3x)°$.
(a) Work out the value of x.
(b) Work out the size of angle ADC.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
$(4x - 5) + (5x + 32) = 180$
$9x + 27 = 180$, so $9x = 153$
$x = 17$
(b) Angle ABC = $3 \times 17 + 0 = 51°$
Angle ADC = $180 - 51 = 129°$
Answer: (a) x = 17 (b) 129°

4 Same gradient on two curves

Two curves have equations $y = x^2 - 3x + 6$ and $y = 2x^2 - 7x + 2$.
Find the value of x at which the curves have the same gradient.

First curve: $\frac{dy}{dx} = 2x - 3$
Second curve: $\frac{dy}{dx} = 4x - 7$
Equal gradients: $2x - 3 = 4x - 7$
$-2x = -4$, so $x = 2$
Answer: x = 2

5 Sample space and conditional probability

A fair 6-sided dice and a spinner numbered 1 to 5 are each thrown once. The first score and the second score are recorded.
(a) Work out the probability that the product is greater than 8.
(b) Given that the sum is odd, work out the probability that the product is greater than 8.

(a) There are $6 \times 5 = 30$ equally likely outcomes, and 15 of them satisfy: the product is greater than 8
Probability = $\frac{1}{2}$
(b) 15 outcomes satisfy: the sum is odd. Of these, 8 also satisfy: the product is greater than 8
Probability = $\frac{8}{15}$
Answer: (a) $\mathbf{\frac{1}{2}}$ **(b)** $\mathbf{\frac{8}{15}}$

6 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = (5x − 10)°, angle ACB = (3x + 28)° and angle CAQ = 36°.
(a) Work out the value of x.
(b) Work out the size of angle BAC.
The diagram is not drawn accurately.

(a) Angle ACB = angle PAB (alternate segment theorem)
3x + 28 = 5x − 10
2x = 38, so x = 19
(b) Angle PAB = 85°
Angles on the straight line PAQ: angle BAC = 180 − 85 − 36 = 59°
Answer: (a) x = 19 (b) 59°

7 Composite functions: fg and gf

f(x) = 4x + 2 and g(x) = x^2 + 6
Solve fg(x) = gf(x)

fg(x) = f(x^2 + 6) = 4x^2 + 26
gf(x) = g(4x + 2) = (4x + 2)^2 + 6 = 16x^2 + 16x + 10
Set them equal and rearrange: 12x^2 + 16x − 16 = 0
Divide by 4 and factorise: (x + 2)(3x − 2) = 0
x = −2 or x = $\frac{2}{3}$

Answer: x = −2 or x = $\frac{2}{3}$

8 Self-inverse function

f(x) = $\frac{4x - 3}{x - k}$, where k is a constant and x ≠ k.

The function f is its own inverse, so f^{-1}(x) = f(x).
Find the value of k.

Let y = $\frac{4x - 3}{x - k}$, so y(x − k) = 4x − 3
xy − ky = 4x − 3, so x(y − 4) = ky − 3
f^{-1}(x) = $\frac{kx - 3}{x - 4}$
For f^{-1}(x) = f(x) the denominators must match: x − 4 = x − k
Answer: k = 4

9 Curves that touch

The curves y = x^2 + 5x − 2 and y = −2x^2 + 23x − 29 touch at one point.
Find the coordinates of this point and the equation of the common tangent.

Equate the two expressions: 3x^2 − 18x + 27 = 0
That is 3(x − 3)^2 = 0 with a repeated root x = 3
y = 22, so the point is (3, 22)
$\frac{dy}{dx}$ = 2x + 5 on the first curve: gradient 11
Tangent: y = 11x − 11
Answer: (3, 22), y = 11x − 11

10 Nested surds

Show that $\sqrt{88 + 30\sqrt{7}} = 5 + 3\sqrt{7}$

Square the right-hand side: $(5 + 3\sqrt{7})^2 = 25 + 30\sqrt{7} + 63$

$= 88 + 30\sqrt{7}$

So $\sqrt{88 + 30\sqrt{7}} = 5 + 3\sqrt{7}$

Answer: $\sqrt{\mathbf{88 + 30\sqrt{7}}} = \mathbf{5 + 3\sqrt{7}}$