Day 249 **IGCSE 10-a-day: Model Solutions**

1 Gradient at a point

Work out the gradient of the curve $y = x^3 + 4x - 1$ at the point where $x = 0$.

$\frac{dy}{dx} = 3x^2 + 4$

When $x = 0$: gradient $= 4$

Answer: 4

2 Arithmetic sequences

The 6th term of an arithmetic sequence is 57.
The 9th term of the sequence is 75.
(a) Find an expression for the nth term of the sequence.
(b) Is 226 a term of the sequence? You must show your working.

(a) Common difference $= (75 - 57) \div (9 - 6) = 6$
nth term $= 6n + c$. For $n = 6$: $6 \times 6 + c = 57$, so $c = 21$
nth term $= 6n + 21$

(b) Solve $6n + 21 = 226$: $6n = 205$, so $n = \frac{205}{6}$

n is not a whole number, so 226 is not a term

Answer: (a) 6n + 21 (b) No

3 Angles with algebra

ABC is an isosceles triangle with AB = AC.
Angle BAC $= (5x - 15)°$
Angle ABC $= (3x + 37)°$
(a) Work out the value of x.
(b) Work out the size of angle BAC.

(a) Angles ABC and ACB are equal, and the angles in a triangle add up to 180°
$(5x - 15) + 2(3x + 37) = 180$
$11x + 59 = 180$, so $11x = 121$
$x = 11$
(b) Angle BAC $= 5 \times 11 - 15 = 40°$

Answer: (a) x = 11 (b) 40°

4 Estimating the median from grouped data

The table gives information about 102 masses.
Work out an estimate for the median.
Assume the values in each class are evenly spread.

Cumulative frequencies: 10, 18, 34, 54, 80, 102
The median is the value at position $102 \div 2 = 51$
It is in the class 30 to 40, with 34 values before it, so we need 17 more of the 20 values in this class

Median $\approx 30 + \frac{17}{20} \times 10 = 38.5$

Answer: 38.5

5 Fencing with a partition

A rectangular paddock is x metres wide and y metres long. It is fenced all round and divided into two equal parts by a fence each of length x metres, parallel to the width.
The total length of fencing used is 60 m.

(a) Show that the area of the paddock is $A = -\frac{3}{2}x^2 + 30x$.

(b) Find the maximum possible area, justifying that it is a maximum.

Fencing: $3x + 2y = 60$, so $y = \frac{60 - 3x}{2}$

$A = xy = \frac{x(60 - 3x)}{2} = -\frac{3}{2}x^2 + 30x$

$\frac{dA}{dx} = -3x + 30 = 0$ gives $x = 10$

$\frac{d^2A}{dx^2} = -3 < 0$, so a maximum

$A = 150$

Answer: 150 m²

6 Area of a triangle given three sides

Work out the area of triangle ABC.
Give your answer correct to 3 significant figures. (Calculator)

$\cos A = \frac{16^2 + 11^2 - 7^2}{2 \times 16 \times 11} = \frac{41}{44}$

$A = 21.280°$

Area = ½ × 16 × 11 × sin 21.280° = 31.9

Answer: 31.9 cm²

7 Histograms with an unmarked scale

The histogram shows the masses of parcels delivered by a courier.
140 of them had a mass between 40 and 60 kg.
(a) How many had a mass between 10 and 20 kg?
(b) How many parcels are there in total?

Frequency is proportional to the area of a bar
40 to 60: bar is 7 squares high and 20 wide, area 140, which represents 140
So 1 unit of area represents 140 ÷ 140 = 1
(a) 10 to 20: 4 × 10 = 40 units of area, so 40 × 1 = 40
(b) Total area = 30 + 40 + 20 + 140 + 70 = 300, so total = 300 × 1 = 300

Answer: (a) 40 (b) 300

8 Bounds of speed, density and pressure

A runner covers a distance of 640 m, correct to 2 significant figures.
The race takes 2.2 s, correct to 1 decimal place.
Work out the upper bound of the speed.
Give your answer correct to 3 significant figures. (Calculator)

$635 \leq$ distance < 645; $2.15 \leq$ time < 2.25
Upper bound of speed = biggest distance ÷ smallest time = 645 ÷ 2.15
= 300.000... = 300 (3 s.f.)

Answer: 300 m/s

9 Probability and quadratic equations

A bag contains x black discs and 3 white discs.
Two discs are taken at random without replacement.

The probability that both are black is $\frac{5}{14}$.

(a) Show that $3x^2 - 13x - 10 = 0$.
(b) Hence work out the number of discs in the bag.

(a) P(both black) = $\frac{x}{x+3} \times \frac{x-1}{x+2} = \frac{5}{14}$

$14\,x(x-1) = 5(x+3)(x+2)$

$14x^2 - 14x = 5x^2 + 25x + 30$ so $9x^2 - 39x - 30 = 0$; divide by 3: $3x^2 - 13x - 10 = 0$

(b) Solve: $x = 5$ or $x = -\frac{2}{3}$

x cannot be negative, so $x = 5$
Number of discs = 5 + 3 = 8
Answer: x = 5, so 8 discs in the bag

10 Normal to y = kx^2: show that

P is the point $(a, 2a^2)$ on the curve $y = 2x^2$, where $a \neq 0$.
The normal to the curve at P meets the y-axis at the point Q.

Show that the y coordinate of Q is $2a^2 + \frac{1}{4}$.

$\frac{dy}{dx} = 4x$, so the gradient of the tangent at P is 4a

Normal gradient = $-\frac{1}{4a}$

Normal: $y - 2a^2 = -\frac{1}{4a}(x - a)$

Put x = 0: $y = 2a^2 + \frac{a}{4a} = 2a^2 + \frac{1}{4}$

Answer: shown: $y = 2a^2 + \frac{1}{4}$