Day 248 **IGCSE 10-a-day: Model Solutions**

1 Coordinates and gradient at the y-intercept

$y = x^4 + x^2 + 5x - 9$

The curve crosses the y-axis at the point A.

Work out the coordinates of A and the gradient of the curve at A.

When $x = 0$, $y = -9$, so A is $(0, -9)$

$\frac{dy}{dx} = 4x^3 + 2x + 5$

When $x = 0$, $\frac{dy}{dx} = 5$

Answer: A = (0, −9), gradient 5

2 Algebraic proof

Prove that $(m + 4)^2 - (m - 4)^2$ is a multiple of 16 for all integer values of m.

$(m + 4)^2 = m^2 + 8m + 16$ and $(m - 4)^2 = m^2 - 8m + 16$

Subtract: $(m + 4)^2 - (m - 4)^2 = 16m$

$16m = 16 \times m$, which is a multiple of 16 because m is an integer

Answer: Proved

3 Venn diagram: shaded region notation

Write down the set notation for the region shaded in the Venn diagram.

Identify which regions are shaded, using the circles E and F

The shaded region is $F \cap E$

Answer: $F \cap E$

4 Successive percentage change

The price of a laptop is decreased by 25%.

The new amount is then decreased by 15%.

Work out the overall percentage change.

Multipliers: 0.75 and 0.85

$0.75 \times 0.85 = 0.6375$

0.6375 is a decrease of 36.25%

Answer: 36.25% decrease

5 Particle at rest

A particle P moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where $s = t^3 - 6t^2 + 9t + 5$ $(t \geq 0)$

Find the acceleration of P at each instant when it is at rest.

$v = \frac{ds}{dt} = 3t^2 - 12t + 9$

P is at rest when $v = 0$: $3t^2 - 12t + 9 = 0$

$3(t - 1)(t - 3) = 0$, so $t = 1$ or $t = 3$

$a = \frac{d^2s}{dt^2} = 6t - 12$

$t = 1$: $a = -6$. $t = 3$: $a = 6$

Answer: -6 m/s^2 when t = 1 and 6 m/s^2 when t = 3

6 Vectors in a triangle

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 3 : 2. N is on AB with AN : NB = 1 : 2.
(a) Find ON in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find MN in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{1}{3}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a}) = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$

(b) OM = $\frac{3}{5}\mathbf{a}$

MN = ON − OM = $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} - \frac{3}{5}\mathbf{a} = \frac{1}{15}\mathbf{a} + \frac{1}{3}\mathbf{b}$

Answer: (a) $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ (b) $\frac{1}{15}\mathbf{a} + \frac{1}{3}\mathbf{b}$

7 Find constants from a turning point

The diagram shows a sketch of the curve $y = ax^2 + bx - 12$.
The curve has a turning point at M(2, −4).
Find the values of a and b.

$\frac{dy}{dx} = 2ax + b = 0$ when $x = 2$, so $b = -4a$
The point (2, −4) is on the curve: $4a + 2b - 12 = -4$
Substitute $b = -4a$: $-4a = 8$, so $a = -2$ and $b = 8$
Answer: $a = -2$, $b = 8$

8 Three sets from information

54 pupils were asked which activities they do.
A: play football B: play netball C: play rugby
n(A) = 25, n(B) = 31, n(C) = 19, n(A ∩ B) = 11, n(A ∩ C) = 10, n(B ∩ C) = 9, n(A ∩ B ∩ C) = 2.
(a) Work out how many are in none of A, B and C.
(b) Work out how many are in exactly one of A, B and C.

n(A ∪ B ∪ C) = 25 + 31 + 19 − 11 − 10 − 9 + 2 = 47
(a) Number in none of the sets = 54 − 47 = 7
(b) Only A = 25 − 11 − 10 + 2 = 6
Only B = 13, only C = 2, so exactly one = 6 + 13 + 2 = 21
Answer: (a) 7 (b) 21

9 Rationalising the denominator

Write $\frac{2+\sqrt{2}}{2-\sqrt{2}}$ in the form $p + q\sqrt{2}$
where p and q are rational numbers.

Multiply top and bottom by $(2 + \sqrt{2})$:
Numerator: $(2 + \sqrt{2})(2 + \sqrt{2}) = 6 + 4\sqrt{2}$
Denominator: 4 − 2 = 2
$= 3 + 2\sqrt{2}$
Answer: $3 + 2\sqrt{2}$

10 Ratio and mixtures

Alloy A holds copper and tin in the ratio 4 : 2.
Alloy B holds copper and tin in the ratio 3 : 2.
12 kg from alloy A is mixed with some of alloy B.
The ratio of copper to tin in the mixture is 8 : 5.
Work out how many kg were taken from alloy B.

Let y kg be taken from B

copper: $8 + \frac{3y}{5}$

tin: $4 + \frac{2y}{5}$

Ratio 8 : 5, so $5(8 + \frac{3y}{5}) = 8(4 + \frac{2y}{5})$

$(\frac{-1}{5})y = -8$, so $y = 40$

Answer: 40 kg