Day 247 **IGCSE 10-a-day: Model Solutions**

1 Product of brackets: tangent gradient

$y = x(x - 2)(2x - 2)$

Work out the gradient of the curve at the point where $x = -2$.

Expand: $y = 2x^3 - 6x^2 + 4x$

$\frac{dy}{dx} = 6x^2 - 12x + 4$

$x = -2$: $\frac{dy}{dx} = 52$

Answer: 52

2 Ordering numbers

Write these numbers in order of size, starting with the largest.

$\frac{4}{7}$, 78%, 86%, $\frac{3}{4}$

Convert each to a decimal: $\frac{4}{7} = 0.571$, 78% = 0.78, 86% = 0.86, $\frac{3}{4} = 0.75$

In order: 86%, 78%, $\frac{3}{4}$, $\frac{4}{7}$

Answer: 86%, 78%, $\frac{3}{4}$, $\frac{4}{7}$

3 Weighted percentages

The ratio of boys to girls in a school is 2 : 3.

80% of the boys and 45% of the girls walk to school.

What percentage of all of these students walk to school?

Imagine 100 students in total: 40 boys and 60 girls

80% of 40 = 32 and 45% of 60 = 27

Together: 59 out of 100

59 ÷ 100 × 100 = 59%

Answer: 59%

4 Factorising fully

Factorise fully $5x^3 - 245x$

Take out the common factor 5x: $5x(x^2 - 49)$

$x^2 - 49$ is a difference of two squares: $(x - 7)(x + 7)$

Answer: $5x(x - 7)(x + 7)$

5 Differentiating brackets with square roots

$y = (\sqrt{x} + 5)^2$

Find $\frac{dy}{dx}$. Give each term of your answer in the form ax^n.

Expand: $y = x + 10x^{1/2} + 25$

$x \to 1$, $10x^{1/2} \to 5x^{-1/2}$

$\frac{dy}{dx} = 1 + 5x^{-1/2}$

Answer: $\frac{dy}{dx} = 1 + 5x^{-1/2}$

6 Express a vector in terms of two vectors

$\mathbf{a} = \begin{pmatrix}1\\1\end{pmatrix}$, $\mathbf{b} = \begin{pmatrix}1\\-4\end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix}8\\-17\end{pmatrix}$.

$\mathbf{c} = m\mathbf{a} + n\mathbf{b}$.

Find the value of m and the value of n.

$m + n = 8$ and $m - 4n = -17$

Multiply the first by (-4) and the second by 1: $-4m - 4n = -32$ and $m - 4n = -17$

Subtract: $-5m = -15$, so $m = 3$

Substitute back: $n = 5$

Answer: $\mathbf{m = 3, n = 5}$

7 Tangents parallel to a line

A curve has equation $y = x^3 - 2x + 3$.

Find the equations of the two tangents to the curve that are parallel to the line $y = x - 2$.

Parallel lines have equal gradients: gradient = 1

$\frac{dy}{dx} = 3x^2 - 2 = 1$, so $3x^2 - 3 = 0$

$3(x + 1)(x - 1) = 0$ gives $x = -1$ or $x = 1$

At $x = -1$: $y = 4$, tangent $y - 4 = 1(x - (-1))$

At $x = 1$: $y = 2$, tangent $y - 2 = 1(x - 1)$

Answer: $\mathbf{y = x + 5}$ and $\mathbf{y = x + 1}$

8 Triangle shapes from position vectors

A = (3, 2), B = (6, 3) and C = (2, 5).

Show that triangle ABC is isosceles and right-angled.

$\mathbf{AB} = \begin{pmatrix}3\\1\end{pmatrix}$ and $\mathbf{AC} = \begin{pmatrix}-1\\3\end{pmatrix}$

$|AB|^2 = 9 + 1 = 10$ and $|AC|^2 = 1 + 9 = 10$

$|AB| = |AC| = \sqrt{10}$, so the triangle is isosceles

$\mathbf{BC} = \begin{pmatrix}-4\\2\end{pmatrix}$ so $|BC|^2 = 16 + 4 = 20$

$|AB|^2 + |AC|^2 = 10 + 10 = 20 = |BC|^2$, so the angle at A is 90° (Pythagoras)

Answer: $|\mathbf{AB}| = |\mathbf{AC}|$ and $\mathbf{AB^2 + AC^2 = BC^2}$

9 Joint and inverse variation

P is directly proportional to v and inversely proportional to the square of t.

When v = 6 and t = 2, P = 36.

Work out the value of P when v = 3 and t = 6.

$P = \frac{kv}{t^2}$

$36 = \frac{k \times 6}{4}$, so $k = 36 \times 4 \div 6 = 24$

$P = \frac{24 \times 3}{36} = 2$

Answer: $\mathbf{P = 2}$

10 Transformations of graphs

The graph of y = f(x) has a minimum point at (−4, 0).
Write down the coordinates of the turning point of the graph of
(a) y = f(x/2)
(b) y = f(x + 4) + 2

(a) Start with the turning point (−4, 0)
Apply the transformation to the point: (−8, 0)
(b) f(x + 4) changes the x-coordinate to −8, and f(x) + 2 changes the y-coordinate to 2
Turning point: (−8, 2)
Answer: (a) (−8, 0) (b) (−8, 2)