Day 246 **IGCSE 10-a-day: Model Solutions**

1 Inverse of a linear function

$f(x) = \frac{3x + 6}{5}$.

(a) Find $f^{-1}(x)$.

(b) Find $f^{-1}(3)$.

(a) Let $y = \frac{3x + 6}{5}$, then rearrange to make x the subject

$x = \frac{5y - 6}{3}$, so $f^{-1}(x) = \frac{5x - 6}{3}$

(b) $f^{-1}(3) = 3$

Answer: (a) $f^{-1}(x) = \frac{5x - 6}{3}$ (b) 3

2 Ordering numbers

Write these numbers in order of size, starting with the smallest.

0.83, $\frac{3}{4}$, $\frac{8}{11}$, 63%

Convert each to a decimal: 0.83 = 0.83, $\frac{3}{4}$ = 0.75, $\frac{8}{11}$ = 0.727, 63% = 0.63

In order: 63%, $\frac{8}{11}$, $\frac{3}{4}$, 0.83

Answer: 63%, $\frac{8}{11}$, $\frac{3}{4}$, 0.83

3 Arc length and sector area

A sector of a circle has radius 4 cm and angle 45°.

(a) Work out the length of the arc of the sector.

(b) Work out the area of the sector. Give each answer in terms of π.

(a) Fraction of the circle = $\frac{45}{360}$

Arc length = $\frac{45}{360} \times 2 \times \pi \times 4 = \pi$ cm

(b) Area = $\frac{45}{360} \times \pi \times 4^2 = 2\pi$ cm^2

Answer: (a) π cm (b) 2π cm^2

4 Collinear points with vectors

$\mathbf{OA} = 3\mathbf{b}$, $\mathbf{OB} = -6\mathbf{a} + \mathbf{b}$ and $\mathbf{OC} = -15\mathbf{a} - 2\mathbf{b}$.

Show that A, B and C lie on a straight line.

$\mathbf{AB} = \mathbf{OB} - \mathbf{OA} = -6\mathbf{a} - 2\mathbf{b}$

$\mathbf{AC} = \mathbf{OC} - \mathbf{OA} = -15\mathbf{a} - 5\mathbf{b}$

$\mathbf{AC} = \frac{5}{2}\mathbf{AB}$

AC is a multiple of AB and they share the point A, so A, B and C are collinear

Answer: $\mathbf{AC} = \frac{5}{2}\mathbf{AB}$, so A, B, C are collinear

5 Particle at rest

A particle P moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where $s = t^3 - 9t^2 + 24t + 2$ $(t \geq 0)$

Find the acceleration of P at each instant when it is at rest.

$v = \frac{ds}{dt} = 3t^2 - 18t + 24$

P is at rest when $v = 0$: $3t^2 - 18t + 24 = 0$

$3(t - 2)(t - 4) = 0$, so $t = 2$ or $t = 4$

$a = \frac{d^2s}{dt^2} = 6t - 18$

$t = 2$: $a = -6$. $t = 4$: $a = 6$

Answer: -6 m/s^2 when t = 2 and 6 m/s^2 when t = 4

6 Perpendicular lines

The diagram shows the straight line L, which passes through the points A and B.

The line M is perpendicular to L and passes through B.

Find the equation of M in the form $y = mx + c$.

Gradient of L $= \frac{-5 - 0}{0 - (-3)} = -\frac{5}{3}$

Perpendicular gradient $= -1 \div (-\frac{5}{3}) = \frac{3}{5}$

M passes through B$(-3, 0)$: $0 = \frac{3}{5} \times (-3) + c$, so $c = \frac{9}{5}$

$y = \frac{3}{5}x + \frac{9}{5}$

Answer: $y = \frac{3}{5}x + \frac{9}{5}$

7 Combined transformations

The point P(5, 3) is reflected in the x-axis.

The image is then reflected in the line $y = x$.

(a) Write down the coordinates of the final image of P.

(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = $(5, -3)$

After the second transformation: P'' = $(-3, 5)$

(b) Follow a general point (x, y) through both transformations: $(x, y) \rightarrow (x, -y) \rightarrow (-y, x)$

The single mapping is $(x, y) \rightarrow (-y, x)$

This is a rotation 90° anticlockwise about the origin

Answer: (a) $(-3, 5)$ (b) Rotation 90° anticlockwise about the origin

8 Product of brackets: tangent gradient

$y = x(x - 4)(2x + 1)$

Work out the gradient of the curve at the point where $x = -2$.

Expand: $y = 2x^3 - 7x^2 - 4x$

$\frac{dy}{dx} = 6x^2 - 14x - 4$

$x = -2$: $\frac{dy}{dx} = 48$

Answer: 48

9 Probability and quadratic equations

A bag contains some sweets. 6 of them are red and the rest are yellow.
Two sweets are taken at random without replacement.

The probability that both are red is $\frac{15}{28}$.

Work out how many sweets are yellow.

Let there be n sweets altogether

P(both red) = $\frac{6}{n} \times \frac{5}{n-1} = \frac{30}{n(n-1)}$

$\frac{30}{n(n-1)} = \frac{15}{28}$, so $n(n-1) = 56$, giving $n^2 - n - 56 = 0$

$(n-8)(n+7) = 0$, so $n = 8$ (n cannot be negative)

Number of sweets that are yellow = $8 - 6 = 2$

Answer: 2

10 Tangent parallel to a given line

The tangents at two points A and B on the curve $y = 2x^3 - 9x^2 + 5x + 4$ are parallel to the line $2y - 10x = 8$.
Find the coordinates of A and B.

Gradient of the line is 5, so $\frac{dy}{dx} = 5$

$6x^2 - 18x + 5 = 5$, so $6x^2 - 18x = 0$ ($\div 6$: $x^2 - 3x = 0$)

$x(x-3) = 0$ so $x = 0$ or $x = 3$

A(0, 4) and B(3, −8)

Answer: (0, 4) and (3, −8)