Day 245 **IGCSE 10-a-day: Model Solutions**

1 Sine rule

The diagram shows triangle ABC. (Calculator)
Work out the length of AC. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{AC}{\sin B} = \frac{BC}{\sin A}$

$\frac{x}{\sin 74°} = \frac{6}{\sin 50°}$

$x = \frac{6 \times \sin 74°}{\sin 50°} = 7.5290$

x = 7.5 cm (1 d.p.)

Answer: AC = 7.5 cm

2 Solve a linear inequality

Solve $-4x + 4 \leq 5x - 18$

Collect x terms: $-9x \leq -22$

Divide by -9 and reverse the inequality sign: $x \geq \frac{22}{9}$

Answer: $x \geq \frac{22}{9}$

3 Increasing for x greater than k

The function $f(x) = -3x^2 + 5$ is decreasing when $x > k$.
Find the value of k.

$f'(x) = -6x$

The turning point is where $f'(x) = 0$: $-6x = 0$

$x = 0$

$f'(x)$ is negative for $x > 0$, so $k = 0$

Answer: k = 0

4 Points satisfying inequalities

R is the region that satisfies all of these inequalities:
$y \leq 3x$ $x + y < 10$ $x > 0$
Which of these points lie in R?
A(0, 1) B(4, 10) C(5, 3) D(8, 4) E(8, 7)

A(0, 1): $y \leq 0$: no; $x + y = 1$: yes; $x = 0$: no
B(4, 10): $y \leq 12$: yes; $x + y = 14$: no; $x = 4$: yes
C(5, 3): $y \leq 15$: yes; $x + y = 8$: yes; $x = 5$: yes
D(8, 4): $y \leq 24$: yes; $x + y = 12$: no; $x = 8$: yes
E(8, 7): $y \leq 24$: yes; $x + y = 15$: no; $x = 8$: yes
In R: C

Answer: C

5 Stationary point of a quartic

$y = -2x^4 + 8x$
Find the coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = -8x^3 + 8$

$\frac{dy}{dx} = 0$: $-8x^3 + 8 = 0$, so $x^3 = 1$

$x = 1$ and $y = 6$

$\frac{d^2y}{dx^2} = -24x^2$

At $x = 1$: $\frac{d^2y}{dx^2} = -24$, so a maximum

Answer: (1, 6), maximum

6 Recurring decimals

Without using a calculator, work out $0.\dot{5} - 0.\dot{0}\dot{5}$
Give your answer as a fraction in its simplest form.

$0.\dot{5} = \frac{5}{9}$

$0.\dot{0}\dot{5} = \frac{5}{99}$

$\frac{5}{9} - \frac{5}{99} = \frac{50}{99}$

Answer: $\mathbf{\frac{50}{99}}$

7 Local maximum and minimum values

$f(x) = 2x^3 - 24x + 2$
Find the y coordinate of the local maximum point and the y coordinate of the local minimum point.

$f'(x) = 6x^2 - 24 = 6(x - 2)(x + 2)$
$f'(x) = 0$ gives $x = -2$ or $x = 2$
$f''(x) = 12x$: $f''(-2) = -24 < 0$ (maximum) and $f''(2) = 24 > 0$ (minimum)
$f(-2) = 34$ and $f(2) = -30$
Answer: Local maximum y = 34, local minimum y = −30

8 Using a line of best fit

The scatter graph shows hours of revision (x) and test mark (y).
The equation of the line of best fit is $y = 2.5x + 7$.
(a) Estimate y when x = 7.
(b) Explain why an estimate of y when x = 25 may be unreliable.

(a) $y = 2.5 \times 7 + 7 = 24.5$
(b) x = 25 is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)
Answer: (a) 24.5 (b) x = 25 is outside the range of the data

9 Cosine rule and area of a triangle

The diagram shows triangle ABC. (Calculator)
(a) Work out angle ACB, marked θ, to 1 decimal place.
(b) Work out the area of triangle ABC to 3 significant figures.
The diagram is not drawn accurately.

(a) $\cos\theta = \frac{9^2 + 12^2 - 12^2}{2 \times 9 \times 12} = \frac{81}{216}$
$\theta = 68.0°$ (1 d.p.)
(b) Area $= \frac{1}{2} \times 9 \times 12 \times \sin 68.0°$
$= 50.1\ \text{cm}^2$ (3 s.f.)
Answer: (a) 68.0° (b) 50.1 cm²

10 Tangency conditions

The line $y = 2x + k$ is a tangent to the circle $x^2 + y^2 = 17$.
Find the possible values of k.
Give any surds in their simplest form.

Substitute $y = 2x + k$: $x^2 + (2x + k)^2 = 17$
Expand: $5x^2 + 4kx + k^2 - 17 = 0$
A tangent touches at one point, so $b^2 - 4ac = 0$: $(4k)^2 - 4 \times 5 \times (k^2 - 17) = 0$
$-4k^2 + 340 = 0$, so $k^2 = 85$
$k = \pm\sqrt{85}$
Answer: k = $\pm\sqrt{85}$