Day 244 **IGCSE 10-a-day: Model Solutions**

1 Differentiating polynomials

Differentiate $6x^3 + 4x - 1$ with respect to x.

Differentiate each term: $x^n \to nx^{n-1}$ and a constant $\to 0$

$6x^3 \to 18x^2$, $4x \to 4$

$\frac{dy}{dx} = 18x^2 + 4$

Answer: $\mathbf{\frac{dy}{dx} = 18x^2 + 4}$

2 Composite functions in x

$f(x) = 2x + 3$ and $g(x) = x^2 + 3x - 1$.

(a) Find fg(x).

(b) Find gf(x).

(a) $fg(x) = f(x^2 + 3x - 1) = 2(x^2 + 3x - 1) + 3$

$= 2x^2 + 6x + 1$

(b) $gf(x) = g(2x + 3) = (2x + 3)^2 + 3(2x + 3) - 1$

$= 4x^2 + 18x + 17$

Answer: (a) $\mathbf{fg(x) = 2x^2 + 6x + 1}$ (b) $\mathbf{gf(x) = 4x^2 + 18x + 17}$

3 Projectile height

A ball is thrown upwards. Its height above the ground, h metres, after t seconds is given by

$h = -5t^2 + 30t + 10$

(a) Work out the greatest height reached by the ball.

(b) Work out the velocity of the ball when t = 4. State whether it is moving up or down.

$\frac{dh}{dt} = -10t + 30 = 0$ gives $t = 3$

$\frac{d^2h}{dt^2} = -10 < 0$ so maximum. $h = 55$

When t = 4: $\frac{dh}{dt} = 30 - 10 \times 4 = -10$

Answer: (a) 55 m (b) −10 m/s, moving down

4 Composite functions

$f(x) = -2x + 4$ and $g(x) = x^2 - 4$

(a) Find gf(x). Give your answer in the form $ax^2 + bx + c$.

(b) Solve $gf(x) = -4$

(a) $gf(x) = g(-2x + 4) = (-2x + 4)^2 - 4$

$= 4x^2 - 16x + 12$

(b) $(-2x + 4)^2 = 0$, so $-2x + 4 = 0$

$x = 2$

Answer: (a) $\mathbf{4x^2 - 16x + 12}$ (b) $\mathbf{x = 2}$

5 Fraction problems

A bag contains counters.

$\frac{1}{3}$ of the counters are red and $\frac{2}{5}$ of the counters are blue.

The rest of the counters, 24 of them, are green.

How many red counters are there?

Fraction that are green = $1 - \frac{1}{3} - \frac{2}{5} = \frac{4}{15}$

$\frac{4}{15}$ of the total is 24

Total = $24 \div \frac{4}{15} = 90$

Red counters = $\frac{1}{3}$ of $90 = 30$

Answer: 30

6 Ratio and algebra

In a coach, the ratio of the number of adults to the number of children is 5 : 3.

Then 15 more adults are added and 19 children are removed.

The ratio is now 11 : 1.

How many children were there at the start?

Let the numbers at the start be 5k and 3k

$(5k + 15) : (3k - 19) = 11 : 1$, so $1(5k + 15) = 11(3k - 19)$

$5k + 15 = 33k - 209$

$-28k = -224$, so $k = 8$

Number of children at the start: $3 \times 8 = 24$

Answer: 24

7 Volume and surface area of composite solids

A solid is made from a cone of height 9 cm and a hemisphere. Both have radius 9 cm.

Work out the total volume of the solid. Give your answer in terms of π.

Cone: $\frac{1}{3} \times \pi \times 9^2 \times 9 = 243\pi$

Hemisphere: $\frac{2}{3} \times \pi \times 9^3 = 486\pi$

Total = $243\pi + 486\pi = 729\pi$ cm^3

Answer: 729π cm^3

8 Independent events and probability rules

A and B are two events.

P(A) = 0.5, P(B) = 0.3 and P(A or B) = 0.65

(a) Work out P(A and B).

(b) Are A and B independent? You must show your working.

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.65 = 0.5 + 0.3 - P(A \cap B)$, so $P(A \cap B) = 0.15$

(b) $P(A) \times P(B) = 0.5 \times 0.3 = 0.15$

This equals $P(A \cap B)$, so A and B are independent

Answer: (a) 0.15 (b) Yes

9 Where two tangents meet

The curve C has equation $y = x^2 - x - 2$.
The points A and B on C have x coordinates -1 and 1.
The tangents to C at A and at B meet at the point T. Find the coordinates of T.

$\frac{dy}{dx} = 2x - 1$. At A: gradient -3, A = $(-1, 0)$
Tangent at A: $y = -3x - 3$. At B: gradient 1, B = $(1, -2)$
Tangent at B: $y = x - 3$
Equate: $-3x - 3 = x - 3$, so $x = 0$ and $y = -3$
Answer: T (0, −3)

10 Frequency density from a table

The table shows the times taken by 171 people.
(a) Work out the frequency density for the class $65 < x \leq 80$.
(b) Estimate how many were greater than 24 minutes.

(a) Frequency density = frequency ÷ class width = 57 ÷ 15 = 3.8
(b) In $20 < x \leq 40$, the part above 24 is 16 out of 20
Estimate = 1.75 × 16 = 28
Add the higher classes: 45 + 28 + 6 + 57, total = 164
Answer: (a) 3.8 (b) 164