Day 243 **IGCSE 10-a-day: Model Solutions**

1 Using a line of best fit

The scatter graph shows distance from town (x) and price (y).
The equation of the line of best fit is $y = -1.5x + 30$.
(a) Estimate y when x = 3.
(b) Explain why an estimate of y when x = 15 may be unreliable.

(a) $y = -1.5 \times 3 + 30 = 25.5$
(b) x = 15 is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)
Answer: (a) 25.5 (b) x = 15 is outside the range of the data

2 Divide then differentiate

$y = \frac{4x^4 + 2x^2 + 4x}{x}$

Find $\frac{dy}{dx}$.

Divide each term by x: $y = 4x^3 + 2x + 4$
$4x^3 \rightarrow 12x^2$, $2x \rightarrow 2$
$\frac{dy}{dx} = 12x^2 + 2$
Answer: $\frac{dy}{dx} = 12x^2 + 2$

3 Find unknown constants in a function

$f(x) = ax^2 + b$, where a and b are constants.
f(1) = 4 and f(3) = −12.
Find the value of a and the value of b.

$a + b = 4$ and $9a + b = -12$
Subtract: $8a = -16$, so $a = -2$
$b = 4 - (-2) \times 1 = 6$
Answer: a = −2, b = 6

4 Averages with algebra

Here are five numbers: 2x + 5, x + 2, x − 2, x + 5, 2x − 1.
The mean of the five numbers is 13.
(a) Work out the value of x.
(b) Work out the median of the five numbers.

(a) Total = (2x + 5) + (x + 2) + (x − 2) + (x + 5) + (2x − 1) = 13 × 5 = 65
7x + 9 = 65, so 7x = 56 and x = 8
The numbers are 21, 10, 6, 13, 15
(b) In order: 6, 10, 13, 15, 21, so the median is 13
Answer: (a) x = 8 (b) 13

5 Estimating the mean from grouped data

The table shows the times taken by 61 people.
(a) Write down the modal class.
(b) Work out an estimate for the mean. Give your answer to 1 decimal place. (Calculator)

(a) Highest frequency is 22: class $40 < x \leq 45$
(b) Midpoints: 22.5, 27.5, 32.5, 37.5, 42.5
$\Sigma fx = 4 \times 22.5 + 9 \times 27.5 + 13 \times 32.5 + 13 \times 37.5 + 22 \times 42.5 = 2182.5$
Mean $\approx 2182.5 \div 61 = 35.8$
Answer: (a) $40 < x \leq 45$ (b) 35.8

6 Prime factors and indices

Write $24^5 \times 30^3$ as a product of powers of prime numbers.

$24 = 2^3 \times 3$, $30 = 2 \times 3 \times 5$

$24^5 = 2^{15} \times 3^5$

$30^3 = 2^3 \times 3^3 \times 5^3$

Add the powers of each prime: $2^{18} \times 3^8 \times 5^3$

Answer: $2^{18} \times 3^8 \times 5^3$

7 Local maximum and minimum values

$f(x) = x^3 - 3x + 9$

Find the y coordinate of the local maximum point and the y coordinate of the local minimum point.

$f'(x) = 3x^2 - 3 = 3(x - 1)(x + 1)$

$f'(x) = 0$ gives $x = -1$ or $x = 1$

$f''(x) = 6x$: $f''(-1) = -6 < 0$ (maximum) and $f''(1) = 6 > 0$ (minimum)

$f(-1) = 11$ and $f(1) = 7$

Answer: Local maximum y = 11, local minimum y = 7

8 Cone made from a sector

A sector of a circle has radius 20 cm and angle 288°.

The sector is made into a cone by joining the two straight edges together.

(a) Work out the radius of the base of the cone.

(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{288}{360} \times 2\pi \times 20 = 32\pi$ cm

This is the circumference of the base: $2\pi r = 32\pi$, so r = 16 cm

(b) Slant height = 20 cm, so height$^2 = 20^2 - 16^2 = 144$, height = 12 cm

Volume = $\frac{1}{3} \times \pi \times 16^2 \times 12 = 1024\pi$ cm^3

Answer: (a) 16 cm (b) 1024π cm^3

9 Cylinder of fixed surface area

A closed cylinder (a container) has radius r cm, height h cm and total surface area 150π cm^2.

(a) Show that its volume, V cm^3, is $V = 75\pi r - \pi r^3$.

(b) Find the maximum value of V in terms of π, and show that h = 2r when V is a maximum.

$2\pi r^2 + 2\pi rh = 150\pi$, so $h = \frac{75 - r^2}{r}$

$V = \pi r^2 h = \pi r(75 - r^2) = 75\pi r - \pi r^3$

$\frac{dV}{dr} = 75\pi - 3\pi r^2 = 0$, so $r^2 = 25$ and r = 5

$\frac{d^2V}{dr^2} = -6\pi r < 0$, so a maximum. $h = \frac{75 - 25}{5} = 10 = 2r$

$V = \pi \times 5^2 \times 10 = 250\pi$

Answer: 250π cm^3

10 Tangency conditions

The line $y = 2x + k$ is a tangent to the circle $x^2 + y^2 = 29$.
Find the possible values of k.
Give any surds in their simplest form.

Substitute $y = 2x + k$: $x^2 + (2x + k)^2 = 29$

Expand: $5x^2 + 4kx + k^2 - 29 = 0$

A tangent touches at one point, so $b^2 - 4ac = 0$: $(4k)^2 - 4 \times 5 \times (k^2 - 29) = 0$

$-4k^2 + 580 = 0$, so $k^2 = 145$

$k = \pm\sqrt{145}$

Answer: $k = \pm\sqrt{145}$