Day 242 **IGCSE 10-a-day: Model Solutions**

1 Factorising fully

Factorise fully $(x + 5)^2 - 36$

This is a difference of two squares: $(x + 5)^2 - 6^2$
$= (x + 5 - 6)(x + 5 + 6)$
$= (x - 1)(x + 11)$
Answer: $(x - 1)(x + 11)$

2 Tangent at a turning point

The graph of $y = -x^2 - 8x - 7$ has a turning point at P.
Find the equation of the tangent to the curve at P.

$\frac{dy}{dx} = -2x - 8$

At P, $\frac{dy}{dx} = 0$, so $-2x - 8 = 0$ and $x = -4$
$y = 9$ at P
The gradient is 0, so the tangent is horizontal: $y = 9$
Answer: $y = 9$

3 Quadrilateral: cosine rule, then area

In quadrilateral ABCD, AB = 7 cm, BC = 8 cm, CD = 9 cm,
angle ABC = 115° and angle ACD = 59°.
Work out the area of ABCD. Give your answer correct to 3 significant figures. (Calculator)

$AC^2 = 7^2 + 8^2 - 2 \times 7 \times 8 \times \cos 115°$ so AC = 12.662 cm
Area ABC = ½ × 7 × 8 × sin 115° = 25.38
Area ACD = ½ × 12.662 × 9 × sin 59° = 48.84
Total area = 25.38 + 48.84 = 74.2
Answer: 74.2 cm^2

4 Exponential growth and decay

A drug in the blood has a mass of 2000 mg. Its mass halves every 5 hours.
(a) Work out the mass after 20 hours.
(b) After how many hours will the mass first be less than 40 mg?

(a) 20 hours is 4 half-lives: $2000 \div 2^4 = 125$ mg
(b) Halve repeatedly: 2000, 1000, 500, 250, 125, 62.5, 31.25
31.25 is the first value below 40, after 6 half-lives, so 6 × 5 = 30 hours
Answer: (a) 125 mg (b) 30 hours

5 Points with a given gradient on y = ax + b/x

A curve has equation $y = -x - \frac{12}{x}$.

Find the coordinates of the points on the curve where the gradient is 2.

$\frac{dy}{dx} = -1 + \frac{12}{x^2}$

Set $\frac{dy}{dx} = 2$: $-3 = \frac{-12}{x^2}$

$x^2 = 4$, so $x = 2$ or $x = -2$
$y = -8$ when $x = 2$ and $y = 8$ when $x = -2$
Answer: $(2, -8)$ and $(-2, 8)$

6 Laws of indices

Given that $\frac{5^3 \times 25^3}{25^2} = 5^n$,

find the value of n.

$25 = 5^2$

Numerator: $5^3 \times 5^6 = 5^9$

Denominator: 5^4

Subtract the powers: $9 - 4 = 5$

Answer: n = 5

7 Area of a segment

The circle has centre O and radius 13 cm. A and B are points on the circle and angle AOB = 100°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{100}{360} \times \pi \times 13^2 = 147.48$ cm^2

Area of triangle OAB = $\frac{1}{2} \times 13 \times 13 \times \sin 100° = 83.216$ cm^2

Segment = sector − triangle = 147.48 − 83.216 = 64.264

= 64.3 cm^2 (3 s.f.)

Answer: 64.3 cm^2

8 Rationalising the denominator

Rationalise the denominator of $\frac{26}{2\sqrt{7}}$

Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{7}$:

$\frac{26}{2\sqrt{7}} = \frac{26\sqrt{7}}{14}$

$= \frac{13\sqrt{7}}{7}$

Answer: $\frac{13\sqrt{7}}{7}$

9 Histogram with unequal class widths

The histogram shows the ages, in years, of some visitors.
The bar for 45 to 55 represents 16 visitors.
Estimate the number of visitors with a value greater than 25.

Frequency = k × height × width. $16 = k \times 4 \times 10$ so $k = \frac{2}{5}$

Part of the 20 to 40 bar: $\frac{2}{5} \times 6 \times (40 - 25) = 36$

Bars above 40: 14 + 16

Total = 66

Answer: 66

10 Minimum surface area of a tank

A tank is a cuboid with a square base of side x metres and height h metres, and has an open top. Its volume is 256 m^3.

(a) Show that the total outside surface area, A m^2, is $A = x^2 + \frac{1024}{x}$.

(b) Find the minimum possible value of A, showing that it is a minimum.

$V = x^2h = 256$, so $h = \frac{256}{x^2}$

$A = x^2 + 4xh = x^2 + 4x \times \frac{256}{x^2} = x^2 + \frac{1024}{x}$

$\frac{dA}{dx} = 2x - 1024x^{-2} = 0$ gives $x^3 = 512$, so $x = 8$

$\frac{d^2A}{dx^2} = 2 + 2048x^{-3} = 6 > 0$, so a minimum

$A = 192$

Answer: 192 m^2