Day 241 **IGCSE 10-a-day: Model Solutions**

1 Differentiating polynomials

Differentiate $5x^5 - 3x^4 + 4x^2 + 5x$ with respect to x.

Differentiate each term: $x^n \to nx^{n-1}$ and a constant $\to 0$

$5x^5 \to 25x^4$, $-3x^4 \to -12x^3$, $4x^2 \to 8x$, $5x \to 5$

$\frac{dy}{dx} = 25x^4 - 12x^3 + 8x + 5$

Answer: $\mathbf{\frac{dy}{dx} = 25x^4 - 12x^3 + 8x + 5}$

2 Standard form: multiplying and dividing

Work out $(3.3 \times 10^{-3}) \times (5.5 \times 10^{1})$.
Give your answer in standard form.

Multiply the numbers: $3.3 \times 5.5 = 18.15$

Add the powers: $10^{-3} \times 10^{1} = 10^{-2}$

$18.15 \times 10^{-2} = 1.815 \times 10^{-1}$

Answer: $\mathbf{1.815 \times 10^{-1}}$

3 Quadratic sequences

Here are the first five terms of a quadratic sequence.
0, 12, 34, 66, 108
Find an expression for the nth term.

First differences: 12, 22, 32, 42

Second differences: 10, 10, 10, so $a = 10 \div 2 = 5$

Subtract $5n^2$ from each term: $-5, -8, -11, -14, -17$

The remaining linear sequence has nth term $-3n - 2$

nth term $= 5n^2 - 3n - 2$

Answer: $\mathbf{5n^2 - 3n - 2}$

4 Similar shapes

Two solid shapes, A and B, are mathematically similar.
The surface area of A is 12 cm^2 and the surface area of B is 108 cm^2.
The volume of A is 200 cm^3.
Work out the volume of B.

Ratio of areas = 12 : 108 = 1 : 9

Ratio of lengths = $\sqrt{1} : \sqrt{9} = 1 : 3$

Ratio of volumes = $1^3 : 3^3$, so scale factor = 27

Volume of B = $200 \times 27 = 5400$ cm^3

Answer: $\mathbf{5400\ cm^3}$

5 Vectors in a triangle with a diagram

In triangle OAB, $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$.
M is on OA so that OM : MA = 3 : 1.
N is on OB so that ON : NB = 1 : 3.
Find $\mathbf{MN}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

$\mathbf{OM} = \frac{3}{4}\mathbf{a}$ and $\mathbf{ON} = \frac{1}{4}\mathbf{b}$

$\mathbf{MN} = \mathbf{MO} + \mathbf{ON} = -\mathbf{OM} + \mathbf{ON}$

$\mathbf{MN} = -\frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$

Answer: $\mathbf{-\frac{3}{4}a + \frac{1}{4}b}$

6 Stationary point with a 1/x term

A curve has equation $y = x^2 + \frac{54}{x}$.

(a) Show that the curve has a stationary point at $x = 3$.

(b) Find the y coordinate of this point and decide whether it is a maximum or a minimum.

$\frac{dy}{dx} = 2x - \frac{54}{x^2}$

When $x = 3$: $\frac{dy}{dx} = 6 - 6 = 0$

$y = 27$

$\frac{d^2y}{dx^2} = 2 + \frac{108}{x^3}$

At $x = 3$: $\frac{d^2y}{dx^2} = 6$, which is positive, so a minimum

Answer: y = 27, minimum

7 Upper and lower bounds

a = 9.6 and b = 2.2,

both correct to 1 decimal place.

Work out the upper bound of a + b.

Each value is rounded, so it can be up to 0.05 above or below

$9.55 \leq a < 9.65$; $2.15 \leq b < 2.25$

Upper bound = 9.65 + 2.25 = 11.9

Answer: 11.9

8 Tangent through a given point: find the constant

A curve has equation $y = -x^2 + 2x + k$, where k is a constant.

The tangent to the curve at the point where $x = -3$ passes through $(-4, -27)$.

Find the value of k.

$\frac{dy}{dx} = -2x + 2$, so at $x = -3$ the gradient is 8

The point of tangency is $(-3, -15 + k)$

Gradient between this point and $(-4, -27)$: $\frac{-27 - (-15 + k)}{-4 - (-3)} = 8$

$-27 - (-15 + k) = -8$

$k = -4$

Answer: k = −4

9 Composite functions

$f(x) = 3x - 2$ and $g(x) = \frac{5}{x}$

(a) Find gf(x).

(b) Solve $gf(x) = -\frac{5}{2}$

(a) $gf(x) = g(3x - 2) = \frac{5}{3x - 2}$

(b) $\frac{5}{3x - 2} = -\frac{5}{2}$, so $3x - 2 = 5 \div -\frac{5}{2} = -2$

$3x = 0$

$x = 0$

Answer: (a) $\frac{5}{3x - 2}$ (b) x = 0

10 Reverse percentage growth and decay

A laptop loses 10% of its value each year.
After 2 years it is worth £1,620.
Work out its value when it was new. (Calculator)

Yearly multiplier = 0.9

Original × 0.9^2 = £1,620

Original = £1,620 ÷ 0.9^2 = £2,000

Answer: £2,000