Day 240 **IGCSE 10-a-day: Model Solutions**

1 Tangent meets the curve again

A curve has equation $y = x^3 + x^2 + 3x - 2$.
The tangent to the curve at the point A, where $x = -1$, meets the curve again at the point B.
Find the coordinates of B.

$\frac{dy}{dx} = 3x^2 + 2x + 3$, so the gradient at A is 4, and A = $(-1, -5)$
Tangent: $y = 4x - 1$
Equate: $x^3 + x^2 - x - 1 = 0$, which has a repeated root $x = -1$
Sum of the roots = -1, so $-1 + (-1) + x = -1$ and $x = 1$
$y = 3$
Answer: B (1, 3)

2 Position vector of a point dividing a line

The position vectors of A and P are $\begin{pmatrix}-3\\3\end{pmatrix}$ and $\begin{pmatrix}3\\12\end{pmatrix}$.
P lies on AB so that AP : PB = 3 : 2.
Find the position vector of B.

$\mathbf{AP} = \begin{pmatrix}3\\12\end{pmatrix} - \begin{pmatrix}-3\\3\end{pmatrix} = \begin{pmatrix}6\\9\end{pmatrix}$

AP is 3 parts, so each part is $\frac{1}{3}$ of $\mathbf{AP} = \begin{pmatrix}2\\3\end{pmatrix}$

AB is 5 parts: $\mathbf{AB} = 5 \times \begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}10\\15\end{pmatrix}$

$\mathbf{OB} = \mathbf{OA} + \mathbf{AB} = \begin{pmatrix}7\\18\end{pmatrix}$

Answer: $\begin{pmatrix}\mathbf{7}\\\mathbf{18}\end{pmatrix}$

3 Square roots as powers

$y = -4x^{5/2} - 3x^2 - \frac{4}{x^{3/2}}$

Find $\frac{dy}{dx}$. Give each term of your answer in the form ax^n.

Write as powers of x: $y = -4x^{5/2} - 3x^2 - 4x^{-3/2}$
$-4x^{5/2} \rightarrow -10x^{3/2}$, $-3x^2 \rightarrow -6x$, $-4x^{-3/2} \rightarrow 6x^{-5/2}$
$\frac{dy}{dx} = -10x^{3/2} - 6x + 6x^{-5/2}$

Answer: $\mathbf{\frac{dy}{dx} = -10x^{3/2} - 6x + 6x^{-5/2}}$

4 Quadratic sequences

Here are the first five terms of a quadratic sequence.
6, 13, 22, 33, 46
Find an expression for the nth term.

First differences: 7, 9, 11, 13
Second differences: 2, 2, 2, so $a = 2 \div 2 = 1$
Subtract n^2 from each term: 5, 9, 13, 17, 21
The remaining linear sequence has nth term $4n + 1$
nth term = $n^2 + 4n + 1$
Answer: $n^2 + 4n + 1$

5 Percentage change

A price is increased by 900%.
By what percentage must the new value be decreased to get back to the original value?

Let the original be 100. After a 900% increase it is 1000.
To return to 100, decrease by 900

Percentage decrease = $\frac{900}{1000} \times 100 = 90\%$

Answer: 90%

6 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = 32° and angle CAQ = 53°.
(a) Work out the size of angle ABC.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angle ABC = angle CAQ = 53° (alternate segment theorem)
(b) Angle ACB = angle PAB = 32° (alternate segment theorem)
Answer: (a) 53° (b) 32°

7 Express a vector in terms of two vectors

$\mathbf{a} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$.
$\mathbf{c} = m\mathbf{a} + n\mathbf{b}$.
Find the value of m and the value of n.

$-2m + 5n = 7$ and $2m + n = -1$
Multiply the first by 1 and the second by 5: $-2m + 5n = 7$ and $10m + 5n = -5$
Subtract: $-12m = 12$, so $m = -1$
Substitute back: $n = 1$
Answer: $m = -1$, $n = 1$

8 Normal meets the curve again (quadratic)

The normal to the curve $y = x^2 + x - 5$ at the point P where x = 0 meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = 2x + 1$: at P, gradient = 1 and $y = -5$
Normal: $y = -x - 5$
Intersection: $x^2 + 2x = 0$
One root is x = 0, and the sum of the roots is −2
So $x_Q = -2 - 0 = -2$
$y = -3$
Answer: (−2, −3)

9 Combined transformations

The point P(−1, 5) is reflected in the x-axis.
The image is then reflected in the line y = x.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (−1, −5)
After the second transformation: P'' = (−5, −1)
(b) Follow a general point (x, y) through both transformations: (x, y) → (x, −y) → (−y, x)
The single mapping is (x, y) → (−y, x)
This is a rotation 90° anticlockwise about the origin
Answer: (a) (−5, −1) (b) Rotation 90° anticlockwise about the origin

10 Estimated mean with an unknown frequency

The table gives information about the times taken by some people to travel to work.
An estimate for the mean is 38.5.
Work out the value of x.

Mid-points: 10, 30, 50, 70
Estimated mean = $\Sigma fm \div \Sigma f$, so $(1210 + 30x) \div (29 + x) = 38.5$
$1210 + 30x = 38.5(29 + x) = 1116.5 + 38.5x$
$(30 - 38.5)x = 1116.5 - 1210$
$-8.5x = -93.5$, so $x = 11$
Answer: x = 11