Day 24 **IGCSE 10-a-day: Model Solutions**

1 Angles of elevation

A point A is on level ground in line with the base of a mast. (Calculator)
The angle of elevation of the top of the mast from A is 31°.
B is a point on the same line, 40 m further from the mast than A.
The angle of elevation of the top of the mast from B is 17°.
Work out the height of the mast. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 31° = \frac{h}{x}$, so $x = h \div \tan 31°$

From B: $\tan 17° = \frac{h}{x + 40}$, so $x + 40 = h \div \tan 17°$

Subtract: $40 = h \div \tan 17° - h \div \tan 31° = h(3.2709 - 1.6643)$
$h = 40 \div 1.6066 = 24.9$ m (1 d.p.)
Answer: 24.9 m

2 Describe a graph transformation

Describe fully the single transformation that maps the graph of y = f(x) onto the graph of
(a) y = f(2x)
(b) y = −f(x)

(a) Every x value is divided by 2
(b) Changing the sign of every y value flips the graph over the x-axis

Answer: (a) A stretch parallel to the x-axis with scale factor $\frac{1}{2}$ (b) A reflection in the x-axis

3 Quadratic formula and surds

Solve $x^2 - 10x + 5 = 0$
Give your answers in the form $a \pm b\sqrt{c}$

Complete the square: $(x - 5)^2 - 25 + 5 = 0$
$(x - 5)^2 = 20$
$x - 5 = \pm\sqrt{20} = \pm 2\sqrt{5}$
$x = 5 \pm 2\sqrt{5}$
Answer: $x = 5 \pm 2\sqrt{5}$

4 Expanding brackets with surds

Show that $(4 + 2\sqrt{2})(4 - 2\sqrt{2})$ is an integer.

Use the difference of two squares: $(4 + 2\sqrt{2})(4 - 2\sqrt{2}) = 16 - (2\sqrt{2})^2$
$= 16 - 8 = 8$
Answer: 8

5 Stationary points and their nature

$y = -2x^3 + 3x^2 + 36x + 3$
(a) Find the coordinates of the stationary points.
(b) Use $\frac{d^2y}{dx^2}$ to decide whether each is a maximum or a minimum.

$\frac{dy}{dx} = -6x^2 + 6x + 36 = -6(x + 2)(x - 3)$
$\frac{dy}{dx} = 0$ gives $x = -2$ or $x = 3$, so the points are $(-2, -41)$ and $(3, 84)$
$\frac{d^2y}{dx^2} = -12x + 6$
$x = -2$: $\frac{d^2y}{dx^2} = 30$, which is positive, so a minimum
$x = 3$: $\frac{d^2y}{dx^2} = -30$, which is negative, so a maximum
Answer: (a) (−2, −41) and (3, 84) (b) (−2, −41) is a minimum, (3, 84) is a maximum

6 Describe a transformation from a diagram

Triangle T is mapped onto triangle T′ by a single transformation.
(a) Describe fully this transformation.
(b) Write down the matrix that represents it.

(a) Compare corresponding vertices, e.g. (1, 3) goes to (−1, 3)
The transformation is a reflection in the y-axis

(b) The columns show where (1, 0) and (0, 1) go: $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

Answer: (a) Reflection in the y-axis (b) $\begin{pmatrix} \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

7 Mixtures and percentage profit

A shop mixes two types of coffee.
Type A costs £14.00 per kg and type B costs £7.00 per kg.
They are mixed in the ratio A : B = 2 : 3.
The mixture is sold for £13.72 per kg.
Work out the percentage profit.

Cost of 5 kg of mixture = 2 × £14.00 + 3 × £7.00 = £49.00
Cost per kg = £49.00 ÷ 5 = £9.80
Profit per kg = £13.72 − £9.80 = £3.92
Percentage profit = 3.92 ÷ 9.80 × 100 = 40%
Answer: 40%

8 Increasing for all x

$f(x) = x^3 + kx^2 + 48x - 2$, where k is a constant.
Given that f is an increasing function for all values of x, find the range of possible values of k.

$f'(x) = 3x^2 + 2kx + 48$
f is increasing for all x when $f'(x) > 0$ for all x, so the quadratic has no real roots
Discriminant: $(2k)^2 - 4 \times 3 \times 48 < 0$, so $4k^2 < 576$
$k^2 < 144$, so $-12 < k < 12$
Answer: $-12 < k < 12$

9 Loci and regions

ABCD is a rectangle with AB = 10 cm and AD = 14 cm.
A point P inside the rectangle is such that AP ≤ 8 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 8 cm: P is inside the quarter circle of radius 8 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves
Area = $\frac{1}{2} \times \frac{1}{4} \times \pi \times 8^2 = 8\pi$ cm^2

Answer: 8π cm^2

10 Area of triangle from a tangent

The tangent to the curve $y = -2x^2 - x - 3$ at the point where $x = 0$ meets the axes at A and B.
Work out the area of triangle OAB, where O is the origin.

$y = -3$ and $\frac{dy}{dx} = -4x - 1$, so the gradient is -1

Tangent: $y = -x - 3$

y-axis: $(0, -3)$. x-axis: $0 = -x - 3$ so $x = -3$

Area $= \frac{1}{2} \times 3 \times 3$

$= \frac{9}{2}$

Answer: $\frac{9}{2}$ square units