Day 239 **IGCSE 10-a-day: Model Solutions**

1 Column vector arithmetic

$\mathbf{a} = \begin{pmatrix} 6 \\ -6 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$.

(a) Work out $\mathbf{a} + \mathbf{b}$.

(b) Work out $3\mathbf{a} - 3\mathbf{b}$.

(a) $\begin{pmatrix} 6 \\ -6 \end{pmatrix} + \begin{pmatrix} 7 \\ -3 \end{pmatrix} = \begin{pmatrix} 13 \\ -9 \end{pmatrix}$

(b) $3\mathbf{a} = \begin{pmatrix} 18 \\ -18 \end{pmatrix}$ and $3\mathbf{b} = \begin{pmatrix} 21 \\ -9 \end{pmatrix}$

$\begin{pmatrix} 18 \\ -18 \end{pmatrix} - \begin{pmatrix} 21 \\ -9 \end{pmatrix} = \begin{pmatrix} -3 \\ -9 \end{pmatrix}$

Answer: (a) $\begin{pmatrix} \mathbf{13} \\ \mathbf{-9} \end{pmatrix}$ **(b)** $\begin{pmatrix} \mathbf{-3} \\ \mathbf{-9} \end{pmatrix}$

2 Where is the second derivative zero?

$y = -x^3 - 9x^2 + 4x - 7$

(a) Find $\frac{d^2y}{dx^2}$.

(b) Find the value of x for which $\frac{d^2y}{dx^2} = 0$, and work out the gradient of the curve at this value of x.

$\frac{dy}{dx} = -3x^2 - 18x + 4$

$\frac{d^2y}{dx^2} = -6x - 18$

$\frac{d^2y}{dx^2} = 0$ gives $-6x - 18 = 0$, so $x = -3$

$\frac{dy}{dx} = 31$ when $x = -3$

Answer: (a) $-6x - 18$ (b) $x = -3$, gradient 31

3 Exact trigonometric ratios from a given ratio

x is an obtuse angle and $\sin x = \frac{7}{10}$.

(a) Find the exact value of cos x.

(b) Find the exact value of tan x. Give each answer as a fraction or a surd. Do not use a calculator.

(a) $\cos^2 x = 1 - (\frac{7}{10})^2 = \frac{51}{100}$

x is obtuse, so cos x is negative: $\cos x = -\frac{\sqrt{51}}{10}$

(b) $\tan x = \frac{\sin x}{\cos x} = \frac{7}{10} \div (-\frac{\sqrt{51}}{10}) = -\frac{7\sqrt{51}}{51}$

Answer: (a) $-\frac{\sqrt{51}}{10}$ **(b)** $-\frac{7\sqrt{51}}{51}$

4 Line and curve intersections

Solve the simultaneous equations

$y = x^2 - 3x + 3$

$y = -2x + 3$

Substitute: $x^2 - 3x + 3 = -2x + 3$

Rearrange: $x^2 - x = 0$

Factorise: $x(x - 1) = 0$, so $x = 0$ or $x = 1$

When $x = 0$: $y = -2 \times 0 + 3 = 3$

When $x = 1$: $y = -2 \times 1 + 3 = 1$

Answer: x = 0, y = 3 and x = 1, y = 1

5 Standard form

Work out $(8 \times 10^4) + (9.8 \times 10^3)$
Give your answer in standard form.

Write the first number as a multiple of 10^3: $8 \times 10^4 = 80 \times 10^3$
$(80 + 9.8) \times 10^3 = 89.8 \times 10^3$
$= 8.98 \times 10^4$
Answer: 8.98×10^4

6 Vertical distance between stationary points

The curve $y = 2x^3 + 6x^2 - 18x - 2$ has two stationary points, A and B.
Work out the vertical distance between A and B.

$\frac{dy}{dx} = 6x^2 + 12x - 18 = 6(x + 3)(x - 1)$
$x = -3$ and $x = 1$
At $x = -3$, $y = 52$; at $x = 1$, $y = -12$
Difference $= 52 - (-12) = 64$
Answer: 64

7 Quadratic sequences

Here are the first five terms of a quadratic sequence.
16, 28, 42, 58, 76
Find an expression for the nth term.

First differences: 12, 14, 16, 18
Second differences: 2, 2, 2, so $a = 2 \div 2 = 1$
Subtract n^2 from each term: 15, 24, 33, 42, 51
The remaining linear sequence has nth term $9n + 6$
nth term $= n^2 + 9n + 6$
Answer: $n^2 + 9n + 6$

8 Self-inverse function

$f(x) = \frac{2x + 3}{x - k}$, where k is a constant and $x \neq k$.

The function f is its own inverse, so $f^{-1}(x) = f(x)$.
Find the value of k.

Let $y = \frac{2x + 3}{x - k}$, so $y(x - k) = 2x + 3$
$xy - ky = 2x + 3$, so $x(y - 2) = ky + 3$
$f^{-1}(x) = \frac{kx + 3}{x - 2}$
For $f^{-1}(x) = f(x)$ the denominators must match: $x - 2 = x - k$
Answer: $k = 2$

9 Normal to y = kx^2: show that

P is the point $(p, 4p^2)$ on the curve $y = 4x^2$, where $p \neq 0$.
The normal to the curve at P meets the y-axis at the point Q.

Show that the y coordinate of Q is $4p^2 + \frac{1}{8}$.

$\frac{dy}{dx} = 8x$, so the gradient of the tangent at P is 8p

Normal gradient $= -\frac{1}{8p}$

Normal: $y - 4p^2 = -\frac{1}{8p}(x - p)$

Put x = 0: $y = 4p^2 + \frac{p}{8p} = 4p^2 + \frac{1}{8}$

Answer: shown: $y = 4p^2 + \frac{1}{8}$

10 Finding the rate of interest

Fatima invested £15,000 in a savings account for 2 years.
Compound interest was paid at a fixed rate per year.
At the end of the 2 years the account held £15,913.50.
Work out the annual rate of interest. (Calculator)

Let the yearly multiplier be m, so $15000 \times m^2 = 15913.50$
$m^2 = 15913.50 \div 15000 = 1.0609$
$m = \sqrt{1.0609} = 1.03$
Rate $= (1.03 - 1) \times 100 = 3\%$
Answer: 3% per year