Day 238 **IGCSE 10-a-day: Model Solutions**

1 Direct variation

F is directly proportional to the cube of s.
When s = 10, F = 750.
(a) Find a formula for F in terms of s.
(b) Find F when s = 4.

$F = ks^3$

$750 = k \times 10^3$ so $k = \frac{3}{4}$

$F = \frac{3}{4}s^3$

$F = \frac{3}{4} \times 64 = 48$

Answer: (a) $F = \frac{3}{4}s^3$ (b) F = 48

2 Equations in disguise

Solve $x - 10\sqrt{x} + 24 = 0$

Let $u = \sqrt{x}$: $u^2 - 10u + 24 = 0$
Factorise: $(u - 4)(u - 6) = 0$, so u = 4 or u = 6
$\sqrt{x} = 4$ or $\sqrt{x} = 6$
x = 16 or x = 36
Answer: x = 16 or x = 36

3 Pressure

A concrete slab in the shape of a cuboid weighs 540 N.
Its smallest face has area 800 cm^2. Its largest face has area 2,000 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the concrete slab can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 800 cm^2 = 0.08 m^2 and 2,000 cm^2 = 0.2 m^2
(a) Greatest pressure is on the smallest face: 540 ÷ 0.08 = 6,750 N/m^2
(b) Least pressure is on the largest face: 540 ÷ 0.2 = 2,700 N/m^2
Answer: (a) 6,750 N/m^2 (b) 2,700 N/m^2

4 Inverse variation

s is inversely proportional to the square of t.
When t = 8, s = 7.03125.
(a) Find a formula for s in terms of t.
(b) Find t when s = 4.5.

$s = \frac{k}{t^2}$

$7.03125 = \frac{k}{8^2}$ so $k = 7.03125 \times 64 = 450$

$s = \frac{450}{t^2}$

$4.5 = \frac{450}{t^2}$ so $t^2 = \frac{450}{4.5} = 100$

t = 10

Answer: (a) $s = \frac{450}{t^2}$ (b) t = 10

5 Trigonometric graphs

The diagram shows the graph of $y = a \sin x + c$.
(a) Write down the value of a and the value of c.
(b) Work out the value of y when $x = 90°$.

(a) The maximum value is 5 and the minimum value is 1
$a = (5 - (1)) \div 2 = 2$
$c = (5 + (1)) \div 2 = 3$
(b) $y = 2 \sin 90° + 3$, and $\sin 90° = 1$
$y = 5$
Answer: (a) a = 2, c = 3 (b) y = 5

6 Intersection and gradient

The curve $y = -x^2 - 5x - 2$ and the line $y = -5x - 6$ intersect at two points A and B, where A has the smaller x coordinate.
(a) Find the coordinates of A and B.
(b) Find the gradient of the curve at A and at B.

$-x^2 - 5x - 2 = -5x - 6$ gives $-x^2 + 4 = 0$, so $-1(x + 2)(x - 2) = 0$
$x = -2$ or $x = 2$, giving A $(-2, 4)$ and B $(2, -16)$
$\frac{dy}{dx} = -2x - 5$: at A it is -1, at B it is -9
Answer: (a) A (−2, 4), B (2, −16) (b) −1 at A, −9 at B

7 Bearings with the sine and cosine rules

A boat leaves a harbour P and travels 114 km on a bearing of 175° to Q. (Calculator)
It then travels 33 km on a bearing of 100° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is 175° + 180° = 355°
Angle PQR = 105° (the difference between the bearings 355° and 100°)
$PR^2 = 114^2 + 33^2 - 2 \times 114 \times 33 \times \cos 105° = 16032$, so PR = 126.6 km
(b) $\cos(QPR) = \frac{114^2 + PR^2 - 33^2}{2 \times 114 \times PR}$, so angle QPR = 14.6°
R is anticlockwise of the direction PQ, so the bearing is 175° − 15° = 160°
Answer: (a) 126.6 km (b) 160°

8 Area of triangle from a tangent

The tangent to the curve $y = 2x^2 + 2x - 4$ at the point where $x = 1$ meets the axes at A and B.
Work out the area of triangle OAB, where O is the origin.

$y = 0$ and $\frac{dy}{dx} = 4x + 2$, so the gradient is 6
Tangent: $y = 6x - 6$
y-axis: $(0, -6)$. x-axis: $0 = 6x - 6$ so $x = 1$
Area $= \frac{1}{2} \times 6 \times 1$
$= 3$
Answer: 3 square units

9 Counting factors

$N = 2^4 \times 5^4$
How many factors does N have?
Include 1 and N.

Add 1 to each power and multiply
$(4 + 1) \times (4 + 1) = 5 \times 5 = 25$
Answer: 25

10 Vertical distance between stationary points

The curve $y = 2x^3 + 3x^2 + 7$ has two stationary points, A and B.
Work out the vertical distance between A and B.

$\frac{dy}{dx} = 6x^2 + 6x = 6x(x + 1)$

$x = -1$ and $x = 0$

At $x = -1$, $y = 8$; at $x = 0$, $y = 7$

Difference $= 8 - 7 = 1$

Answer: 1