**Day 237** **IGCSE 10-a-day: Model Solutions**

## 1 Differentiating polynomials

Differentiate $-2x^5 - 5x^2 - 4$ with respect to x.

Differentiate each term: $x^n \to nx^{n-1}$ and a constant $\to 0$

$-2x^5 \to -10x^4$, $-5x^2 \to -10x$

$\frac{dy}{dx} = -10x^4 - 10x$

**Answer:** $\mathbf{\frac{dy}{dx} = -10x^4 - 10x}$

## 2 Composite functions

f(x) = 3x + 1 and g(x) = $x^2$ + 7

(a) Find gf(x). Give your answer in the form $ax^2 + bx + c$.

(b) Solve gf(x) = 8

(a) gf(x) = g(3x + 1) = $(3x + 1)^2 + 7$

$= 9x^2 + 6x + 8$

(b) $(3x + 1)^2 = 1$, so $3x + 1 = \pm 1$

$3x + 1 = 1$ or $3x + 1 = -1$

$x = -\frac{2}{3}$ or x = 0

**Answer: (a)** $\mathbf{9x^2 + 6x + 8}$ **(b)** $\mathbf{x = -\frac{2}{3}}$ **or x = 0**

## 3 Composite functions in x

f(x) = $-4x + 7$ and g(x) = $x^2 + x + 5$.

(a) Find fg(x).

(b) Find gf(x).

(a) fg(x) = f($x^2 + x + 5$) = $-4(x^2 + x + 5) + 7$

$= -4x^2 - 4x - 13$

(b) gf(x) = g($-4x + 7$) = $(-4x + 7)^2 + (-4x + 7) + 5$

$= 16x^2 - 60x + 61$

**Answer: (a) fg(x) =** $\mathbf{-4x^2 - 4x - 13}$ **(b) gf(x) =** $\mathbf{16x^2 - 60x + 61}$

## 4 Reverse percentage growth and decay

A car loses 8% of its value each year.

After 2 years it is worth £846.40.

Work out its value when it was new. (Calculator)

Yearly multiplier = 0.92

Original × $0.92^2$ = £846.40

Original = £846.40 ÷ $0.92^2$ = £1,000

**Answer: £1,000**

## 5 Vectors in a triangle

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 3 : 2. N is on AB with AN : NB = 2 : 3.
(a) Find ON in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find MN in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{2}{5}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{2}{5}(\mathbf{b} - \mathbf{a}) = \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$

(b) OM = $\frac{3}{5}\mathbf{a}$

MN = ON − OM = $\frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b} - \frac{3}{5}\mathbf{a} = \frac{2}{5}\mathbf{b}$

**Answer: (a) $\frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$ (b) $\frac{2}{5}\mathbf{b}$**

## 6 Cylinder of fixed surface area

A closed cylinder (a drum) has radius r cm, height h cm and total surface area $384\pi$ cm$^2$.
(a) Show that its volume, V cm$^3$, is $V = 192\pi r - \pi r^3$.
(b) Find the maximum value of V in terms of $\pi$, and show that h = 2r when V is a maximum.

$2\pi r^2 + 2\pi rh = 384\pi$, so $h = \frac{192 - r^2}{r}$

$V = \pi r^2 h = \pi r(192 - r^2) = 192\pi r - \pi r^3$

$\frac{dV}{dr} = 192\pi - 3\pi r^2 = 0$, so $r^2 = 64$ and $r = 8$

$\frac{d^2V}{dr^2} = -6\pi r < 0$, so a maximum. $h = \frac{192 - 64}{8} = 16 = 2r$

$V = \pi \times 8^2 \times 16 = 1024\pi$

**Answer: $1024\pi$ cm$^3$**

## 7 Exponential graphs

The diagram shows the graph of $y = ka^x$, where k and a are constants and a > 0.
The curve passes through the points (2, 12) and (4, 48).
Find the value of k and the value of a.

Substitute the points: $k\,a^2 = 12$ and $k\,a^4 = 48$

Divide: $a^2 = \frac{48}{12} = 4$

a = 2 (positive, since a > 0)
From the first point: $k \times 4 = 12$, so k = 3
**Answer: k = 3, a = 2**

## 8 Tangents parallel to a line

A curve has equation $y = x^3 - 3x^2 + 7x + 4$.
Find the equations of the two tangents to the curve that are parallel to the line y = 7x + 6.

Parallel lines have equal gradients: gradient = 7

$\frac{dy}{dx} = 3x^2 - 6x + 7 = 7$, so $3x^2 - 6x = 0$

$3x(x - 2) = 0$ gives x = 0 or x = 2
At x = 0: y = 4, tangent $y - 4 = 7(x - 0)$
At x = 2: y = 14, tangent $y - 14 = 7(x - 2)$
**Answer: y = 7x + 4 and y = 7x**

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## 9 Surds and fractions

Simplify $\frac{1}{3+\sqrt{7}} + \frac{1}{3-\sqrt{7}}$

Give your answer with a rational denominator.

Common denominator: $(3+\sqrt{7})(3-\sqrt{7}) = 9 - 7 = 2$

Numerator: $(3-\sqrt{7}) + (3+\sqrt{7})$

$= 6$

$\frac{6}{2} = 3$

**Answer: 3**

## 10 Transformed curve from a diagram

The diagram shows the graph of y = f(x) with a minimum point at (−2, 2).
The dashed curve C is the image of y = f(x) after a translation.
The minimum point of C is (−5, −1).
Write down the equation of C in terms of f.

The minimum point has moved from (−2, 2) to (−5, −1)

Translation by $\begin{pmatrix} -3 \\ -3 \end{pmatrix}$

x moves left by 3: f(x + 3). y moves down by 3: − 3

**Answer: y = f(x + 3) − 3**