Day 236 **IGCSE 10-a-day: Model Solutions**

1 Bearings with the sine and cosine rules

A boat leaves a harbour P and travels 117 km on a bearing of 250° to Q. (Calculator)
It then travels 108 km on a bearing of 205° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is 250° − 180° = 070°
Angle PQR = 135° (the difference between the bearings 070° and 205°)
$PR^2 = 117^2 + 108^2 - 2 \times 117 \times 108 \times \cos 135° = 43223$, so PR = 207.9 km

(b) $\cos(QPR) = \frac{117^2 + PR^2 - 108^2}{2 \times 117 \times PR}$, so angle QPR = 21.6°

R is anticlockwise of the direction PQ, so the bearing is 250° − 22° = 228°

Answer: (a) 207.9 km (b) 228°

2 Simplifying surds

Write $\sqrt{28} + \sqrt{112} + \sqrt{7}$ in the form $a\sqrt{7}$
where a is an integer.

$\sqrt{28} = 2\sqrt{7}$; $\sqrt{112} = 4\sqrt{7}$
$(2 + 4 + 1)\sqrt{7} = 7\sqrt{7}$

Answer: $7\sqrt{7}$

3 Local maximum and minimum values

$f(x) = x^3 - 3x - 1$
Find the y coordinate of the local maximum point and the y coordinate of the local minimum point.

$f'(x) = 3x^2 - 3 = 3(x - 1)(x + 1)$
$f'(x) = 0$ gives $x = -1$ or $x = 1$
$f''(x) = 6x$: $f''(-1) = -6 < 0$ (maximum) and $f''(1) = 6 > 0$ (minimum)
$f(-1) = 1$ and $f(1) = -3$

Answer: Local maximum y = 1, local minimum y = −3

4 Simplifying algebraic fractions

Simplify fully $\frac{3x^2 - 27}{x^2 + 5x - 24}$

Factorise the top: $3(x - 3)(x + 3)$
Factorise the bottom: $(x - 3)(x + 8)$
Cancel the common factor $(x - 3)$: $\frac{3(x + 3)}{x + 8}$

Answer: $\frac{3(x + 3)}{x + 8}$

5 Coordinates and gradient at the y-intercept

$y = 3x^3 - 6x^2 - 3x + 9$
The curve crosses the y-axis at the point A.
Work out the coordinates of A and the gradient of the curve at A.

When x = 0, y = 9, so A is (0, 9)
$\frac{dy}{dx} = 9x^2 - 12x - 3$
When x = 0, $\frac{dy}{dx} = -3$

Answer: A = (0, 9), gradient −3

6 3D trigonometry: cuboid

ABCDEFGH is a cuboid. AB = 9 cm, BC = 11 cm and CG = 10 cm.
(a) Work out the length of AG.
(b) Work out the size of the angle between AG and the face ABFE.
Give answers to 3 significant figures or 1 decimal place. (Calculator)

(a) $AG^2 = 9^2 + 11^2 + 10^2 = 302$, so AG = 17.38 cm
(b) G projects onto F on face ABFE (FG is perpendicular to the face), so the angle is GAF
$AF^2 = 9^2 + 10^2 = 181$ and FG = 11

$\tan GAF = \frac{11}{\sqrt{181}}$, angle = 39.3°

Answer: (a) 17.4 cm (b) 39.3°

7 Angles of elevation

A point A is on level ground in line with the base of a mast. (Calculator)
The angle of elevation of the top of the mast from A is 41°.
B is a point on the same line, 25 m further from the mast than A.
The angle of elevation of the top of the mast from B is 19°.
Work out the height of the mast. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 41° = \frac{h}{x}$, so $x = h \div \tan 41°$

From B: $\tan 19° = \frac{h}{x + 25}$, so $x + 25 = h \div \tan 19°$

Subtract: $25 = h \div \tan 19° - h \div \tan 41° = h(2.9042 - 1.1504)$
h = 25 ÷ 1.7538 = 14.3 m (1 d.p.)

Answer: 14.3 m

8 Stationary point of y = ax + b/√x

A curve has equation $y = 5x + \frac{270}{\sqrt{x}}$ for x > 0.

Find the coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 5 - 135x^{-3/2}$

$\frac{dy}{dx} = 0$: $135x^{-3/2} = 5$, so $x^{3/2} = 27$

$x = 27^{2/3} = 9$, and y = 135

$\frac{d^2y}{dx^2} = \frac{405}{2}x^{-5/2}$

At x = 9, $\frac{d^2y}{dx^2} = \frac{5}{6}$, which is positive, so a minimum

Answer: (9, 135), minimum

9 Probability from a frequency tree

A factory makes 5,000 parts. 2% of them are faulty.
A scanner flags 98% of faulty parts and wrongly flags 1% of parts that are not faulty.
A part is chosen at random from the parts flagged by the scanner.
Work out the probability that this part is faulty.

2% of 5,000 = 100 are faulty and 4,900 are not
Flagged and genuine: 100 × 98% = 98
Flagged but not: 4,900 × 1% = 49
Total flagged or positive = 98 + 49 = 147

Probability = 98 ÷ 147 = $\frac{2}{3}$

Answer: $\frac{2}{3}$

10 Variation: percentage change

y is inversely proportional to x.
x is decreased by 50%.
Work out the percentage change in y.

x is multiplied by 0.5

y is multiplied by $\frac{1}{0.5^1}$

Multiplier for y = 2
Percentage increase = 100%
Answer: 100% increase