Day 235 **IGCSE 10-a-day: Model Solutions**

1 Substitution

Work out the value of $\sqrt{y^2 - 4xz}$
when x = 2, y = 1 and z = −3.

Substitute the values: $\sqrt{1^2 - 4 \times 2 \times (-3)}$
$= \sqrt{25}$
$= 5$
Answer: 5

2 Sample space and conditional probability

A spinner numbered 1 to 4 and a spinner numbered 1 to 5 are each thrown once. The first score and the second score are recorded.
(a) Work out the probability that the sum is greater than 7.
(b) Given that at least one number is 3, work out the probability that the sum is greater than 7.

(a) There are 4 × 5 = 20 equally likely outcomes, and 3 of them satisfy: the sum is greater than 7
Probability = $\frac{3}{20}$

(b) 8 outcomes satisfy: at least one number is 3. Of these, 1 also satisfy: the sum is greater than 7
Probability = $\frac{1}{8}$

Answer: (a) $\frac{3}{20}$ (b) $\frac{1}{8}$

3 Error intervals

The speed of a cyclist is 51 km/h, truncated to a whole number.
Write down the error interval for the speed of a cyclist, using s.

Truncating cuts off the later digits without rounding.
The smallest possible value is 51 and the value must be less than 52
$51 \leq s < 52$
Answer: $51 \leq s < 52$

4 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the length of the chord AB.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AM^2 = OA^2 - OM^2 = 25^2 - 7^2 = 576$
AM = 24 cm
AB = 2 × 24 = 48 cm

(b) sin(angle AOM) = $\frac{24}{25}$
Angle AOM = 73.74°, so angle AOB = 2 × 73.74 = 147.5°
Answer: (a) 48 cm (b) 147.5°

5 Tangent to a factorised cubic

A curve has equation y = (x + 1)(x + 1)(x + 2).
Find the equation of the tangent to the curve at the point where x = 2.

Expand: $y = x^3 + 4x^2 + 5x + 2$
$\frac{dy}{dx} = 3x^2 + 8x + 5$
At x = 2: y = 36 and gradient = 33
$y - 36 = 33(x - 2)$
$y = 33x - 30$
Answer: $y = 33x - 30$

6 Using a quadratic nth term

The nth term of a sequence is $n^2 - 2$.
(a) Work out the 8th term.
(b) One term of the sequence is 119. Work out the value of n for this term.

(a) $1 \times 8^2 + 0 \times 8 - 2 = 62$
(b) $n^2 - 2 = 119$ so $n^2 - 121 = 0$
Solve (factorise or use the formula): $n = 11$ or $n = -11$. n must be a positive integer, so $n = 11$
Answer: (a) 62 (b) n = 11

7 Minimum running cost

The cost, C pence per kilometre, of running a van at a steady speed of v km/h is modelled by
$C = \frac{1}{10}v + \frac{160}{v}$ $(v > 0)$
Find the speed at which the cost per kilometre is least, and show that it is a minimum.
Work out the least cost per kilometre.

$\frac{dC}{dv} = \frac{1}{10} - 160v^{-2} = 0$, so $v^2 = 1600$ and $v = 40$
$\frac{d^2C}{dv^2} = 320v^{-3} = \frac{1}{200} > 0$, so a minimum
$C = \frac{160}{40} + \frac{40}{10} = 8$
Answer: v = 40 km/h, C = 8 pence per km

8 Mixtures and percentage profit

A shop mixes two types of tea.
Type A costs £12.00 per kg and type B costs £14.00 per kg.
They are mixed in the ratio A : B = 5 : 3.
The mixture is sold for £17.85 per kg.
Work out the percentage profit.

Cost of 8 kg of mixture = $5 \times £12.00 + 3 \times £14.00 = £102.00$
Cost per kg = $£102.00 \div 8 = £12.75$
Profit per kg = $£17.85 - £12.75 = £5.10$
Percentage profit = $5.10 \div 12.75 \times 100 = 40\%$
Answer: 40%

9 A squared in terms of A and I

$A = \begin{pmatrix} -2 & 6 \\ -2 & 1 \end{pmatrix}$.
$A^2 = pA + qI$, where I is the 2×2 identity matrix.
Find the value of p and the value of q.

$A^2 = \begin{pmatrix} -8 & -6 \\ 2 & -11 \end{pmatrix}$
Top right entries: $-6 = p \times 6$, so $p = -1$
Top left entries: $-8 = -1 \times (-2) + q$, so $q = -10$
Check bottom right: $-1 \times 1 - 10 = -11$
Answer: p = −1, q = −10

10 Fastest rate of growth

The number of bacteria in a culture, N, after t hours is modelled by

$N = -t^3 + 15t^2 + 5t + 200$ for $0 \leq t \leq 20$

Find the time at which N is increasing most rapidly, and the rate of increase at this time.

$\frac{dN}{dt} = -3t^2 + 30t + 5$

Rate is greatest when $\frac{d^2N}{dt^2} = -6t + 30 = 0$, so $t = 5$

$\frac{d^2N}{dt^2}$ is decreasing (gradient -6), so the rate is a maximum

$\frac{dN}{dt} = 80$ when $t = 5$

Answer: t = 5, rate 80 per hour