Day 234 **IGCSE 10-a-day: Model Solutions**

1 Stationary points of a cubic

A curve has equation $y = 2x^3 + 3x^2 - 36x + 5$.
Find the coordinates of the stationary points of the curve.

$\frac{dy}{dx} = 6x^2 + 6x - 36$

Stationary points: $\frac{dy}{dx} = 0$, so $6(x + 3)(x - 2) = 0$

$x = -3$ or $x = 2$
$y = 86$ when $x = -3$ and $y = -39$ when $x = 2$
Answer: $(-3, 86)$ and $(2, -39)$

2 Equations with fractions

Solve $\frac{x + 3}{8} - \frac{x + 5}{2} = -4$

Multiply every term by 8: $(x + 3) - 4(x + 5) = -32$
Expand: $-3x - 17 = -32$
$-3x = -15$
$x = 5$
Answer: $x = 5$

3 Stratified sampling

The table shows the number of students in each of four groups.
(a) A stratified sample of 38 is taken. Work out how many come from Year 9.
(b) A different stratified sample has 16 from Year 7. Work out the size of this sample.

Total = 200 + 225 + 275 + 250 = 950

(a) Year 9: $\frac{275}{950} \times 38 = 11$

(b) Fraction sampled = $16 \div 200 = \frac{2}{25}$, so sample size = $\frac{2}{25} \times 950 = 76$

Answer: (a) 11 (b) 76

4 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 2 cm, DB = 4 cm and the area of triangle ADE is 4 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 2 + 4 = 6 cm
Length scale factor = $6 \div 2 = 3$
Area scale factor = $(3)^2 = 9$
Area of triangle ABC = $4 \times 9 = 36$ cm^2
Trapezium = $36 - 4 = 32$ cm^2
Answer: 32 cm^2

5 Fractional power functions

$f(x) = -6x^{-1/2} + 4x^{-3/2}$
Find $f'(x)$ and work out the value of $f'(4)$.

$-6x^{-1/2} \rightarrow 3x^{-3/2}$, $4x^{-3/2} \rightarrow -6x^{-5/2}$
$f'(x) = 3x^{-3/2} - 6x^{-5/2}$
$f'(4)$: $\sqrt{4} = 2$, so $f'(4) = \frac{3}{16}$
Answer: $f'(x) = 3x^{-3/2} - 6x^{-5/2}$, $f'(4) = \frac{3}{16}$

6 Reverse percentages

The value of a house is increased by 8% in one year.
The next year its value is increased by 10%.
It is now worth £297,000.
Work out the value of the house before the first change.

Multipliers: 1.08, 1.1
Overall multiplier = 1.08 × 1.1 = 1.188
Original = final ÷ multiplier = £297,000 ÷ 1.188
= £250,000
Answer: £250,000

7 Equations in disguise

Solve $x - 13\sqrt{x} + 42 = 0$

Let $u = \sqrt{x}$: $u^2 - 13u + 42 = 0$
Factorise: $(u - 6)(u - 7) = 0$, so $u = 6$ or $u = 7$
$\sqrt{x} = 6$ or $\sqrt{x} = 7$
$x = 36$ or $x = 49$
Answer: $x = 36$ or $x = 49$

8 Range and inverse of a quadratic

$g(x) = x^2 + 4x + 10$ for $x \leq -2$.
(a) Find the range of g.
(b) Find $g^{-1}(x)$.

(a) $g(x) = (x + 2)^2 + 6$
$x \leq -2$ includes the minimum point, so $g(x) \geq 6$
(b) $y = (x + 2)^2 + 6$, so $(x + 2)^2 = y - 6$
$x \leq -2$ means $x + 2$ is negative or zero, so $x = -2 - \sqrt{y - 6}$
$g^{-1}(x) = -2 - \sqrt{x - 6}$
Answer: (a) $g(x) \geq 6$ (b) $g^{-1}(x) = -2 - \sqrt{x - 6}$

9 Normal meets the x-axis

The diagram shows the curve $y = -x^2 - 4x + 6$ and the normal at P(1, 1), meeting the x-axis at Q. R is (1, 0).
(a) Find the normal in the form $x + ky = c$.
(b) Work out the area of triangle PQR.

$\frac{dy}{dx} = -2x - 4$, so the gradient of the tangent at P is -6

Normal gradient = $\frac{1}{6}$

$y - 1 = \frac{1}{6}(x - 1)$. Multiply by 6: $x - 6y = -5$

Q: $y = 0$ gives $x = -5$, so Q = $(-5, 0)$ and QR = 6
Area = ½ × 6 × 1 = 3
Answer: (a) $x - 6y = -5$ (b) 3

10 Arithmetic sequence from two terms

An arithmetic sequence has term 4 equal to -1 and term 9 equal to 19.
(a) Find an expression for the nth term.
(b) Find the first term of the sequence that is greater than 250.

(a) Common difference = $(19 - (-1)) \div 5 = 4$
nth term = $4n + c$; $-1 = 4 \times 4 + c$ so $c = -17$
nth term = $4n - 17$
(b) $4n - 17 > 250$: $n > 66.75$
Smallest whole number $n = 67$; the term is 251
Answer: (a) $4n - 17$ (b) 251