Day 233 **IGCSE 10-a-day: Model Solutions**

1 Negative powers

$y = -5x^4 + x - \frac{5}{x}$

Find $\frac{dy}{dx}$.

Write negative powers as indices: $y = -5x^4 + x - 5x^{-1}$

$-5x^4 \to -20x^3$, $x \to 1$, $-5x^{-1} \to 5x^{-2}$

$\frac{dy}{dx} = -20x^3 + 1 + 5x^{-2}$

Answer: $\mathbf{\frac{dy}{dx} = -20x^3 + 1 + \frac{5}{x^2}}$

2 Finding a percentage rate

A car was bought for £7,100.
Each year its value decreases by the same percentage.
After 2 years it is worth £4,544.
Work out the percentage decrease each year.

Multiplier for 2 years = £4,544 ÷ £7,100 = 0.64
Yearly multiplier = the square root of 0.64 = 0.8
20% decrease each year
Answer: 20%

3 Turning point of a quadratic by differentiation

$f(x) = 2x^2 - 16x + 7$
Use differentiation to find the coordinates of the turning point of the graph of y = f(x). State whether it is a maximum or a minimum.

$f'(x) = 4x - 16$
$f'(x) = 0$: $4x - 16 = 0$, so $x = 4$
$f(4) = -25$
$f''(x) = 4$, which is positive, so it is a minimum
Answer: (4, −25), minimum

4 Integers satisfying a double inequality

n is an integer.
How many values of n satisfy
$2 \le -2n + 1 < 21$

Subtract 1 from each part: $1 \le -2n < 20$

Divide each part by −2 (reverse the signs): $-10 < n \le -\frac{1}{2}$

Integers: −9, −8, −7, −6, −5, −4, −3, −2, −1
Count: 9
Answer: 9

5 Comparing box plots

The box plots show the times taken by two groups, A and B.
(a) Work out the interquartile range for each.
(b) Compare the distributions of A and B.

(a) IQR of A = 80 − 35 = 45
IQR of B = 65 − 50 = 15
(b) Group A has the lower median (55 compared with 60), so on average Group A was faster
Group B has the smaller interquartile range (15 compared with 45), so its times are more consistent
Answer: (a) A: 45, B: 15 (b) A has the lower median, B has the smaller IQR

6 Composite functions

f(x) = −2x + 5 and g(x) = x^2 + 3

(a) Find gf(x). Give your answer in the form ax^2 + bx + c.

(b) Solve gf(x) = 228

(a) gf(x) = g(−2x + 5) = $(-2x + 5)^2$ + 3

= $4x^2$ − 20x + 28

(b) $(-2x + 5)^2$ = 225, so −2x + 5 = ±15

−2x + 5 = 15 or −2x + 5 = −15

x = −5 or x = 10

Answer: (a) $4x^2$ − 20x + 28 (b) x = −5 or x = 10

7 Sine rule: finding a side

Work out the length of AC.

Give your answer correct to 3 significant figures. (Calculator)

Angle B = 180° − 64° − 48° = 68°

$$\frac{AC}{\sin 68°} = \frac{11.8}{\sin 64°}$$

$$AC = \frac{11.8 \times \sin 68°}{\sin 64°} = 12.2$$

Answer: 12.2 cm

8 Vector proof: collinear points

OA = **a** and OB = **b**.

OP = −**a** − **b**, OQ = 8**a** − 13**b** and OR = 11**a** − 17**b**.

(a) Show that P, Q and R lie on a straight line.

(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = 9**a** − 12**b**

QR = OR − OQ = 3**a** − 4**b**

QR = $\frac{1}{3}$ PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line

(b) PQ : QR = 3 : 1

Answer: (b) 3 : 1

9 Find unknown coefficients from gradients

The curve y = ax^2 + bx + 1 has gradient −3 when x = 0 and gradient 5 when x = 1.

Find the values of a and b.

$$\frac{dy}{dx} = 2ax + b$$

x = 0: b = −3

x = 1: 2a + b = 5

Subtract and solve: a = 4

b = −3

Answer: a = 4, b = −3

10 Loci and regions

ABCD is a rectangle with AB = 10 cm and AD = 11 cm.
A point P inside the rectangle is such that AP ≤ 10 cm and P is nearer to A than to B.
Work out the area of the region in which P can lie. Give your answer in terms of π and surds.

Nearer to A than to B: P is on the A side of the perpendicular bisector of AB, a distance 5 cm from A
AP ≤ 10 cm: P is inside the quarter circle of radius 10 cm centred at A

The bisector meets the arc where $\cos\theta = \frac{5}{10}$, so $\theta = 60°$ at A. The region is a right-angled triangle plus a sector of angle 30°

Triangle: base 5 cm, height $10\sin 60° = 5\sqrt{3}$ cm, area $= \frac{1}{2} \times 5 \times 5\sqrt{3} = \frac{25}{2}\sqrt{3}$

Sector of angle 30° : $\frac{30}{360} \times \pi \times 10^2 = \frac{25}{3}\pi$

Area $= \frac{25}{2}\sqrt{3} + \frac{25}{3}\pi$ cm^2

Answer: $\left(\frac{25}{2}\sqrt{3} + \frac{25}{3}\pi\right)$ cm^2