Day 232 **IGCSE 10-a-day: Model Solutions**

1 Standard form: multiplying and dividing

Work out the square root of 2.25×10^2.
Give your answer in standard form.

Write 2.25×10^2 as 2.25×10^2 (even power of 10)
Square root: $\sqrt{2.25} = 1.5$ and $\sqrt{(10^2)} = 10^1$
$= 1.5 \times 10^1$
Answer: 1.5×10^1

2 Similar shapes

Two statues are mathematically similar.
The heights of the statues are 8 cm and 28 cm.
The smaller one has volume 200 cm^3.
Work out the volume of the larger one.

Scale factor of lengths $= 28 \div 8 = \frac{7}{2}$

Scale factor of volumes $= (\frac{7}{2})^3 = \frac{343}{8}$

$200 \times \frac{343}{8} = 8575\ cm^3$

Answer: 8575 cm^3

3 Point with a given gradient (quadratic)

A curve has equation $y = 3x^2 + 2x - 8$.
The gradient at the point P on the curve is -4.
Find the coordinates of P.

$\frac{dy}{dx} = 6x + 2$
$6x + 2 = -4$, so $6x = -6$
$x = -1$
$y = -7$
Answer: $(-1, -7)$

4 Income tax

In one country, income tax is worked out like this:
The first £12,500 of income is not taxed.
The next £35,000 is taxed at 20%.
All income above that is taxed at 45%.
Beth earns £82,500 in a year.
Work out how much income tax Beth pays.

Taxed at 20%: £35,000
20% of £35,000 = £7,000
Taxed at 45%: £82,500 − £12,500 − £35,000 = £35,000
45% of £35,000 = £15,750
Total tax = £7,000 + £15,750 = £22,750
Answer: £22,750

5 Pressure

A stone block in the shape of a cuboid weighs 720 N.
Its smallest face has area 800 cm^2. Its largest face has area 2,500 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the stone block can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 800 cm^2 = 0.08 m^2 and 2,500 cm^2 = 0.25 m^2
(a) Greatest pressure is on the smallest face: $720 \div 0.08 = 9{,}000\ N/m^2$
(b) Least pressure is on the largest face: $720 \div 0.25 = 2{,}880\ N/m^2$
Answer: (a) 9,000 N/m^2 (b) 2,880 N/m^2

6 Fibonacci-type sequences

A sequence is defined by $u_{n+2} = u_{n+1} + u_n$.

$u_1 = 7$ and $u_2 = x$.

(a) Show that $u_5 = 3x + 14$.

(b) Given that $u_5 = 41$, find x and hence work out u_8.

(a) $u_3 = 7 + x$, $u_4 = u_3 + u_2 = 2x + 7$, $u_5 = u_4 + u_3 = 3x + 14$

(b) $3x + 14 = 41$ so $3x = 27$ and $x = 9$

Terms: 7, 9, 16, 25, 41, 66, 107, 173

$u_8 = 173$

Answer: $x = 9$, $u_8 = 173$

7 Functions that are their own inverse

$f(x) = \frac{4x + 5}{x - 4}$ where $x \neq 4$

(a) Show that $ff(x) = x$.

(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by $(x - 4)$

$ff(x) = \frac{4(4x + 5) + 5(x - 4)}{(4x + 5) - 4(x - 4)}$

Top $= 16x + 20 + 5x - 20 = 21x$

Bottom $= 4x + 5 - 4x + 16 = 21$

$ff(x) = \frac{21x}{21} = x$

(b) $ff(x) = x$, so f is its own inverse: $f^{-1}(x) = \frac{4x + 5}{x - 4}$

Answer: (b) $f^{-1}(x) = \frac{4x + 5}{x - 4}$

8 Intersection and gradient

The curve $y = -x^2 + x - 2$ and the line $y = -5x + 6$ intersect at two points A and B, where A has the smaller x coordinate.

(a) Find the coordinates of A and B.

(b) Find the gradient of the curve at A and at B.

$-x^2 + x - 2 = -5x + 6$ gives $-x^2 + 6x - 8 = 0$, so $-1(x - 2)(x - 4) = 0$

$x = 2$ or $x = 4$, giving A $(2, -4)$ and B $(4, -14)$

$\frac{dy}{dx} = -2x + 1$: at A it is -3, at B it is -7

Answer: (a) A $(2, -4)$, B $(4, -14)$ (b) -3 at A, -7 at B

9 Surds and fractions

Simplify $\frac{2}{5 + \sqrt{10}} - \frac{3}{5 - \sqrt{10}}$

Give your answer with a rational denominator.

Common denominator: $(5 + \sqrt{10})(5 - \sqrt{10}) = 25 - 10 = 15$

Numerator: $2(5 - \sqrt{10}) - 3(5 + \sqrt{10})$

$= -5 - 5\sqrt{10}$

$\frac{-5 - 5\sqrt{10}}{15} = -\frac{1 + \sqrt{10}}{3}$

Answer: $-\frac{1 + \sqrt{10}}{3}$

10 Increasing and decreasing with powers

$f(x) = x + \frac{4}{x}$ for $x > 0$

(a) Find the range of values of x for which f is a decreasing function.
(b) Write down the least value of f(x).

$f'(x) = 1 - 4x^{-2} = 1 - \frac{4}{x^2}$

f is decreasing when $f'(x) < 0$: $1 < \frac{4}{x^2}$, so $x^2 < 4$

With $x > 0$ this gives $0 < x < 2$
f decreases then increases, so the least value is at $x = 2$: $f(2) = 4$
Answer: (a) $0 < x < 2$ (b) 4