Day 231 **IGCSE 10-a-day: Model Solutions**

1 Differentiating polynomials

Find $\frac{dy}{dx}$ when $y = x^5 - 3x^3 - 7$

Differentiate each term: $x^n \to nx^{n-1}$ and a constant $\to 0$

$x^5 \to 5x^4$, $-3x^3 \to -9x^2$

$\frac{dy}{dx} = 5x^4 - 9x^2$

Answer: $\mathbf{\frac{dy}{dx} = 5x^4 - 9x^2}$

2 Simplifying algebraic fractions

Simplify fully $\frac{5x^2 - 20}{x^2 + 5x - 14}$

Factorise the top: $5(x - 2)(x + 2)$

Factorise the bottom: $(x - 2)(x + 7)$

Cancel the common factor $(x - 2)$: $\frac{5(x + 2)}{x + 7}$

Answer: $\mathbf{\frac{5(x + 2)}{x + 7}}$

3 Ratio and expressions

$x : y = 4 : 3$

Work out the ratio $(5x - 6y) : (3x + 6y)$

Give your answer in its simplest form.

$x : y = 4 : 3$, so let $x = 4t$ and $y = 3t$

$5(4t) - 6(3t) = 2t$

$3(4t) + 6(3t) = 30t$

$2t : 30t = 1 : 15$

Answer: 1 : 15

4 Gradient from a tangent

The diagram shows part of the curve $y = -x^2 + 3x - 3$. The line shown is the tangent to the curve at the point P. (Not drawn accurately.)

(a) Use the two marked points on the tangent to work out its gradient.

(b) Use differentiation to find the x coordinate of P.

Gradient $= \frac{-2 - 1}{3 - 0} = -1$

$\frac{dy}{dx} = -2x + 3$

$-2x + 3 = -1$, so $x = 2$

Answer: (a) -1 (b) $x = 2$

5 Powers of standard form

Work out $(4.2 \times 10^4)^2$

Give your answer in standard form.

$(4.2 \times 10^4)^2 = 4.2^2 \times (10^4)^2$

$4.2^2 = 17.64$ and $(10^4)^2 = 10^8$

$17.64 \times 10^8 = 1.764 \times 10^9$

Answer: 1.764×10^9

6 Tangent to a factorised cubic

A curve has equation $y = (x + 3)(x + 3)(x - 1)$.
Find the equation of the tangent to the curve at the point where $x = -1$.

Expand: $y = x^3 + 5x^2 + 3x - 9$
$\frac{dy}{dx} = 3x^2 + 10x + 3$
At $x = -1$: $y = -8$ and gradient $= -4$
$y + 8 = -4(x + 1)$
$y = -4x - 12$
Answer: $y = -4x - 12$

7 Finding a percentage rate

A machine was bought for £4,900.
Each year its value decreases by the same percentage.
After 2 years it is worth £2,401.
Work out the percentage decrease each year.

Multiplier for 2 years = £2,401 ÷ £4,900 = 0.49
Yearly multiplier = the square root of 0.49 = 0.7
30% decrease each year
Answer: 30%

8 Extreme value of an expression in a region

x and y satisfy these inequalities:
$x \geq 2$, $y \geq 2$, $x + y \leq 9$
Find the greatest possible value of $5x + 3y$.

The region is a triangle. The extreme value occurs at a vertex.
Vertices: (2, 2), (7, 2), (2, 7)
Values of $5x + 3y$: 16, 41, 31
Greatest value = 41
Answer: 41

9 Trigonometric graphs

The diagram shows the graph of $y = a \sin x + c$.
(a) Write down the value of a and the value of c.
(b) Work out the value of y when $x = 330°$.

(a) The maximum value is 5 and the minimum value is -3
$a = (5 - (-3)) \div 2 = 4$
$c = (5 + (-3)) \div 2 = 1$

(b) $y = 4 \sin 330° + 1$, and $\sin 330° = -\frac{1}{2}$

$y = -1$
Answer: (a) $a = 4$, $c = 1$ (b) $y = -1$

10 Intersecting lines in a triangle

$\mathbf{OA} = \mathbf{a}$, $\mathbf{OB} = \mathbf{b}$, $\mathbf{OP} = \frac{1}{3}\mathbf{a}$ and $\mathbf{OQ} = \frac{1}{4}\mathbf{b}$.

AQ and BP meet at X.
(a) Find $\mathbf{OX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find the ratio AX : XQ.

Let $\mathbf{AX} = s\mathbf{AQ}$. Then $\mathbf{OX} = \mathbf{a} + s(-\mathbf{a} + \frac{1}{4}\mathbf{b}) = (1 - s)\mathbf{a} + \frac{1}{4}s\mathbf{b}$

Let $\mathbf{BX} = t\mathbf{BP}$. Then $\mathbf{OX} = \mathbf{b} + t(\frac{1}{3}\mathbf{a} - \mathbf{b}) = \frac{1}{3}t\mathbf{a} + (1 - t)\mathbf{b}$

Compare coefficients: $1 - s = \frac{1}{3}t$ and $\frac{1}{4}s = 1 - t$

Solving gives $s = \frac{8}{11}$ and $t = \frac{9}{11}$

(a) $\mathbf{OX} = \frac{3}{11}\mathbf{a} + \frac{2}{11}\mathbf{b}$

(b) $AX = \frac{8}{11} \times AQ$, so $AX : XQ = \frac{8}{11} : \frac{3}{11} = 8 : 3$

Answer: (a) $\frac{3}{11}\mathbf{a} + \frac{2}{11}\mathbf{b}$ (b) 8 : 3