Day 230 **IGCSE 10-a-day: Model Solutions**

1 Find constants from a turning point

The diagram shows a sketch of the curve $y = ax^2 + bx + 19$.
The curve has a turning point at M(−3, −8).
Find the values of a and b.

$\frac{dy}{dx} = 2ax + b = 0$ when $x = -3$, so $b = 6a$

The point (−3, −8) is on the curve: $9a - 3b + 19 = -8$

Substitute $b = 6a$: $-9a = -27$, so $a = 3$ and $b = 18$

Answer: a = 3, b = 18

2 Standard form: multiplying and dividing

Work out $(4.7 \times 10^1) \times (7.9 \times 10^8)$.
Give your answer in standard form.

Multiply the numbers: $4.7 \times 7.9 = 37.13$

Add the powers: $10^1 \times 10^8 = 10^9$

$37.13 \times 10^9 = 3.713 \times 10^{10}$

Answer: 3.713×10^{10}

3 Product of brackets: tangent gradient

$y = x(x - 4)(2x - 2)$
Work out the gradient of the curve at the point where $x = 1$.

Expand: $y = 2x^3 - 10x^2 + 8x$

$\frac{dy}{dx} = 6x^2 - 20x + 8$

$x = 1$: $\frac{dy}{dx} = -6$

Answer: −6

4 Solving trigonometric equations

Solve $\sin x = -\frac{1}{2}$ for $0° \leq x \leq 360°$.

Do not use a calculator.

The related acute angle satisfies $\sin \theta = \frac{1}{2}$, so $\theta = 30°$

sine is negative in the 3rd and 4th quadrants, so $x = 180° + 30°$ and $x = 360° - 30°$
$x = 210°$ and $x = 330°$

Answer: x = 210° or x = 330°

5 Fractional power functions

$f(x) = -4x^{3/2} + 3x - 6x^{1/2}$
Find $f'(x)$ and work out the value of $f'(4)$.

$-4x^{3/2} \to -6x^{1/2}$, $3x \to 3$, $-6x^{1/2} \to -3x^{-1/2}$

$f'(x) = -6x^{1/2} + 3 - 3x^{-1/2}$

$f'(4)$: $\sqrt{4} = 2$, so $f'(4) = -\frac{21}{2}$

Answer: $f'(x) = -6x^{1/2} + 3 - 3x^{-1/2}$, $f'(4) = -\frac{21}{2}$

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6 Independent events and probability rules

A and B are two events.
P(A) = 0.5, P(B) = 0.8 and P(A or B) = 0.9
(a) Work out P(A and B).
(b) Are A and B independent? You must show your working.

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.9 = 0.5 + 0.8 - P(A \cap B)$, so $P(A \cap B) = 0.4$
(b) $P(A) \times P(B) = 0.5 \times 0.8 = 0.4$
This equals $P(A \cap B)$, so A and B are independent
Answer: (a) 0.4 (b) Yes

7 Standard form problems

A atom has a diameter of 5×10^{-8} metres.
Atoms are placed in a straight line, touching, to make a length of 2.45×10^{-5} metres.
How many atoms are in the line?
Give your answer in standard form.

Number = total length ÷ diameter
$(2.45 \times 10^{-5}) \div (5 \times 10^{-8})$
$2.45 \div 5 = 0.49$ and $10^{-5} \div 10^{-8} = 10^{3}$
$= 4.9 \times 10^{2}$
Answer: 4.9×10^{2}

8 Range and inverse of a quadratic

$g(x) = x^2 + 2x + 3$ for $x \geq -1$.
(a) Find the range of g.
(b) Find $g^{-1}(x)$.

(a) $g(x) = (x + 1)^2 + 2$
$x \geq -1$ includes the minimum point, so $g(x) \geq 2$
(b) $y = (x + 1)^2 + 2$, so $(x + 1)^2 = y - 2$
$x \geq -1$ means $x + 1$ is positive or zero, so $x = -1 + \sqrt{y - 2}$
$g^{-1}(x) = -1 + \sqrt{x - 2}$
Answer: (a) $g(x) \geq 2$ (b) $g^{-1}(x) = -1 + \sqrt{x - 2}$

9 Circle and line intersections

Solve the simultaneous equations
$x^2 + y^2 = 50$
$y = 2x - 5$

Substitute into the first equation: $x^2 + (2x - 5)^2 = 50$
Expand: $5x^2 - 20x - 25 = 0$
Divide by 5: $x^2 - 4x - 5 = 0$
Factorise: $(x + 1)(x - 5) = 0$, so $x = -1$ or $x = 5$
$y = 2x - 5$ gives $y = -7$ and $y = 5$
Answer: $x = -1$, $y = -7$ and $x = 5$, $y = 5$

10 Winning a match

Leila and Uma play a match. The first to win 2 tennis sets wins the match.

The probability that Leila wins any one of the tennis sets is $\frac{3}{4}$.

Work out the probability that Leila wins the match.

Leila wins by WW (2 games) or WLW or LWW (3 games), where W means Leila wins that game

P(WW) = $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

P(WLW) = P(LWW) = $\frac{3}{4} \times \frac{1}{4} \times \frac{3}{4} = \frac{9}{64}$

Total = $\frac{9}{16} + 2 \times \frac{9}{64} = \frac{27}{32}$

Answer: $\mathbf{\frac{27}{32}}$