Day 23 **IGCSE 10-a-day: Model Solutions**

1 Equations with fractions

Solve $\frac{x-3}{5} + \frac{x-7}{2} = -2$

Multiply every term by 10: $2(x-3) + 5(x-7) = -20$
Expand: $7x - 41 = -20$
$7x = 21$
$x = 3$
Answer: x = 3

2 Negative powers

Differentiate $6x^2 + 2x + \frac{7}{x^2}$ with respect to x.

Write negative powers as indices: $y = 6x^2 + 2x + 7x^{-2}$
$6x^2 \rightarrow 12x$, $2x \rightarrow 2$, $7x^{-2} \rightarrow -14x^{-3}$
$\frac{dy}{dx} = 12x + 2 - 14x^{-3}$
Answer: $\frac{dy}{dx} = 12x + 2 - \frac{14}{x^3}$

3 Arithmetic sequence from two terms

An arithmetic sequence has term 4 equal to 12 and term 8 equal to 0.
(a) Find an expression for the nth term.
(b) Find the first term of the sequence that is less than −100.

(a) Common difference = $(0 - 12) \div 4 = -3$
nth term = $-3n + c$; $12 = (-3) \times 4 + c$ so $c = 24$
nth term = $-3n + 24$

(b) $-3n + 24 < -100$: $n < \frac{124}{3}$
Smallest whole number n = 42; the term is −102
Answer: (a) −3n + 24 (b) −102

4 Similar shapes

Two shapes are mathematically similar.
The smaller has height 6 cm and area 36 cm^2.
The larger has area 225 cm^2.
Work out the height of the larger shape.

Ratio of areas = 225 : 36 = 25 : 4
Ratio of lengths = 5 : 2
Height = $6 \times \frac{5}{2} = 15$ cm
Answer: 15 cm

5 Moving averages

The table shows the daily visitors over six periods.
The 3-point moving average centred on Thu is 17.
(a) Work out the value of x.
(b) Work out the 3-point moving average centred on Fri.

(a) $(22 + x + 22) \div 3 = 17$
$22 + x + 22 = 51$, so $x = 7$
(b) $(7 + 22 + 40) \div 3 = 69 \div 3 = 23$
Answer: (a) x = 7 (b) 23

6 Shaded areas: exact answers

The diagram shows a circle inside a square of side 8 cm.
The circle touches all four sides of the square.
Work out the area of the shaded region. Give your answer in terms of π.

Area of square = $8 \times 8 = 64$
Radius of circle = $8 \div 2 = 4$ cm
Area of circle = $\pi \times 4^2 = 16\pi$
Shaded area = $64 - 16\pi$ cm^2
Answer: $(64 - 16\pi)$ cm^2

7 Exact values from a given ratio

θ is an acute angle and $\sin\theta = \frac{5}{7}$.

Find the exact value of (a) $\cos\theta$ (b) $\tan\theta$.
Give each answer as a fraction, rationalising any denominator.

Right-angled triangle: opposite = 5, hypotenuse = 7
Adjacent = $\sqrt{7^2 - 5^2} = \sqrt{24}$
(a) $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{2\sqrt{6}}{7}$
(b) $\tan\theta = \frac{5}{\sqrt{24}} = \frac{5\sqrt{6}}{12}$
Answer: (a) $\frac{2\sqrt{6}}{7}$ (b) $\frac{5\sqrt{6}}{12}$

8 Area of triangle from a tangent

The tangent to the curve $y = x^2 - 3x + 6$ at the point where $x = 2$ meets the axes at A and B.
Work out the area of triangle OAB, where O is the origin.

$y = 4$ and $\frac{dy}{dx} = 2x - 3$, so the gradient is 1
Tangent: $y = x + 2$
y-axis: (0, 2). x-axis: $0 = x + 2$ so $x = -2$
Area = $\frac{1}{2} \times 2 \times 2$
$= 2$
Answer: 2 square units

9 Surds and fractions

Simplify $\frac{3}{3+\sqrt{7}} - \frac{1}{3-\sqrt{7}}$
Give your answer with a rational denominator.

Common denominator: $(3+\sqrt{7})(3-\sqrt{7}) = 9 - 7 = 2$
Numerator: $3(3-\sqrt{7}) - (3+\sqrt{7})$
$= 6 - 4\sqrt{7}$
$\frac{6-4\sqrt{7}}{2} = 3 - 2\sqrt{7}$
Answer: $3 - 2\sqrt{7}$

10 Square roots as powers

$y = 2x^{5/2} + x^2 - \frac{4}{\sqrt{x}}$

Find $\frac{dy}{dx}$. Give each term of your answer in the form ax^n.

Write as powers of x: $y = 2x^{5/2} + x^2 - 4x^{-1/2}$

$2x^{5/2} \rightarrow 5x^{3/2}$, $x^2 \rightarrow 2x$, $-4x^{-1/2} \rightarrow 2x^{-3/2}$

$\frac{dy}{dx} = 5x^{3/2} + 2x + 2x^{-3/2}$

Answer: $\mathbf{\frac{dy}{dx} = 5x^{3/2} + 2x + 2x^{-3/2}}$