Day 229 **IGCSE 10-a-day: Model Solutions**

1 Second derivative

$y = x^3 + 2x^2 - 6x$

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(b) Work out the value of $\frac{d^2y}{dx^2}$ when $x = 3$.

$\frac{dy}{dx} = 3x^2 + 4x - 6$

$\frac{d^2y}{dx^2} = 6x + 4$

$x = 3$: $\frac{d^2y}{dx^2} = 22$

Answer: (a) $\frac{dy}{dx} = 3x^2 + 4x - 6$, $\frac{d^2y}{dx^2} = 6x + 4$ (b) 22

2 Adding and scaling matrices

$A = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $B = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$.

(a) Work out A + B.

(b) Work out 2A − 3B.

(a) $A + B = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

(b) $2A = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and $3B = \begin{pmatrix} -12 \\ 3 \end{pmatrix}$

$2A - 3B = \begin{pmatrix} 14 \\ -3 \end{pmatrix}$

Answer: (a) $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$ (b) $\begin{pmatrix} 14 \\ -3 \end{pmatrix}$

3 Expanding three brackets

Expand and simplify $(x + 2)^2(x - 3)$

Expand the first two brackets: $(x + 2)^2 = x^2 + 4x + 4$

Multiply by $(x - 3)$: $x(x^2 + 4x + 4) - 3(x^2 + 4x + 4)$

$= x^3 + x^2 - 8x - 12$

Answer: $x^3 + x^2 - 8x - 12$

4 Minimum surface area of a tank

A tank is a cuboid with a square base of side x metres and height h metres, and has a closed top. Its volume is 125 m^3.

(a) Show that the total outside surface area, A m^2, is $A = 2x^2 + \frac{500}{x}$.

(b) Find the minimum possible value of A, showing that it is a minimum.

$V = x^2h = 125$, so $h = \frac{125}{x^2}$

$A = 2x^2 + 4xh = 2x^2 + 4x \times \frac{125}{x^2} = 2x^2 + \frac{500}{x}$

$\frac{dA}{dx} = 4x - 500x^{-2} = 0$ gives $x^3 = 125$, so $x = 5$

$\frac{d^2A}{dx^2} = 4 + 1000x^{-3} = 12 > 0$, so a minimum

$A = 150$

Answer: 150 m^2

5 Compound interest

Beth puts £3,500 in a savings account.
Compound interest is paid at 4% in year 1, 3% in year 2 and 5% in year 3.
Work out the amount in the account at the end of year 3.
Give your answer to the nearest penny. (Calculator)

Multipliers: 1.04, 1.03, 1.05
£3,500 × 1.04 × 1.03 × 1.05
= £3,936.66
Answer: £3,936.66

6 Nature from the gradient either side

$y = -2x^3 - 6x^2 - 5$
(a) Show that there is a stationary point at $x = 0$.
(b) By finding the gradient when $x = -\frac{1}{2}$ and when $x = \frac{1}{2}$, decide whether it is a maximum or a minimum.

$\frac{dy}{dx} = -6x^2 - 12x$, so when $x = 0$: $\frac{dy}{dx} = 0$

$x = -\frac{1}{2}$: gradient $= \frac{9}{2}$ (positive)

$x = \frac{1}{2}$: gradient $= -\frac{15}{2}$ (negative)

The gradient goes from positive to 0 to negative, so $(0, -5)$ is a maximum
Answer: (b) maximum

7 Rationalising the denominator

Write $\frac{12}{2+\sqrt{2}}$ in the form $p + q\sqrt{2}$
where p and q are integers.

Multiply top and bottom by $(2 - \sqrt{2})$:
$\frac{12}{2+\sqrt{2}} = \frac{12(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})}$
Denominator: $4 - 2 = 2$
$= 12 - 6\sqrt{2}$
Answer: $12 - 6\sqrt{2}$

8 At least one

The probability that the 8:15 train is late on any day is $\frac{1}{5}$.

Events on different occasions are independent.
Work out the probability that this happens at least once in 3 days.
Give your answer correct to 3 significant figures. (Calculator)

P(never) $= (1 - \frac{1}{5})^3 = \frac{4}{5}^3 = 0.512$
P(at least once) = 1 − P(never) = 1 − 0.512
= 0.488
Answer: 0.488

9 Loci and regions

ABCD is a rectangle with AB = 2 cm and AD = 6 cm.
A point P inside the rectangle is such that AP ≤ 2 cm and P is nearer to A than to B.
Work out the area of the region in which P can lie. Give your answer in terms of π and surds.

Nearer to A than to B: P is on the A side of the perpendicular bisector of AB, a distance 1 cm from A
AP ≤ 2 cm: P is inside the quarter circle of radius 2 cm centred at A

The bisector meets the arc where $\cos\theta = \frac{1}{2}$, so $\theta = 60°$ at A. The region is a right-angled triangle plus a sector of angle 30°

Triangle: base 1 cm, height $2\sin 60° = 1\sqrt{3}$ cm, area $= \frac{1}{2} \times 1 \times 1\sqrt{3} = \frac{1}{2}\sqrt{3}$

Sector of angle 30° : $\frac{30}{360} \times \pi \times 2^2 = \frac{1}{3}\pi$

Area $= \frac{1}{2}\sqrt{3} + \frac{1}{3}\pi$ cm^2

Answer: $(\frac{1}{2}\sqrt{3} + \frac{1}{3}\pi)$ cm^2

10 Compound interest: find the rate

£10,000 is invested for 2 years at r% per year compound interest.
At the end of 2 years the investment is worth £12,544.
Work out the value of r. (Calculator)

$10000 \times (1 + \frac{r}{100})^2 =$ £12,544

$(1 + \frac{r}{100})^2 = 12544 \div 10000 = 1.2544$

$1 + \frac{r}{100} = 1.12$

r = 12

Answer: r = 12