Day 228 **IGCSE 10-a-day: Model Solutions**

1 Multiplying and dividing surds

Simplify fully $\frac{2\sqrt{150}}{3\sqrt{6}}$

$\sqrt{150} = 5\sqrt{6}$

$\frac{2\sqrt{150}}{3\sqrt{6}} = \frac{10\sqrt{6}}{3\sqrt{6}}$

$= \frac{10}{3}$

Answer: $\mathbf{\frac{10}{3}}$

2 Substitution

Work out the value of $a^2 - 3bc$
when $a = 1$, $b = 2$ and $c = 3$.

Substitute the values: $1^2 - 3 \times 2 \times 3$

$= 1 - 18$

$= -17$

Answer: $\mathbf{-17}$

3 Point with a given gradient (quadratic)

A curve has equation $y = -2x^2 - 5x - 5$.
The gradient at the point P on the curve is -1.
Find the coordinates of P.

$\frac{dy}{dx} = -4x - 5$

$-4x - 5 = -1$, so $-4x = 4$

$x = -1$

$y = -2$

Answer: $\mathbf{(-1, -2)}$

4 Solve a linear inequality

Solve $\frac{x - 2}{3} \leq \frac{5 + 4x}{4}$

Multiply by 12: $4(x - 2) \leq 3(5 + 4x)$

Expand and collect: $-8x \leq 23$

Divide by -8 and reverse the inequality sign: $x \geq -\frac{23}{8}$

Answer: $\mathbf{x \geq -\frac{23}{8}}$

5 Combining means

The mean of the heights in cm for the Year 10 students is 61.
The mean of the heights in cm for the Year 11 students is 51.
The mean for all of the Year 10 students and Year 11 students together is 54.
Work out the ratio of the number of Year 10 students to the number of Year 11 students.

Let there be p Year 10 students and q Year 11 students

Total = $61p + 51q$, and the overall mean gives $61p + 51q = 54(p + q)$

$7p = 3q$

$p : q = 3 : 7$

Answer: 3 : 7

6 Three stationary points of a quartic

$y = x^4 - 8x^2 + 7$

Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 4x^3 - 16x = 4x(x - 2)(x + 2)$

$\frac{dy}{dx} = 0$ gives $x = -2$, $x = 0$ or $x = 2$

Points: $(-2, -9)$, $(0, 7)$, $(2, -9)$

$\frac{d^2y}{dx^2} = 12x^2 - 16$

$x = -2$: $\frac{d^2y}{dx^2} = 32$; $x = 0$: $\frac{d^2y}{dx^2} = -16$; $x = 2$: $\frac{d^2y}{dx^2} = 32$

So $(-2, -9)$ is a minimum, $(0, 7)$ is a maximum, $(2, -9)$ is a minimum

Answer: $(-2, -9)$ minimum; $(0, 7)$ maximum; $(2, -9)$ minimum

7 Terms of a quadratic sequence

The nth term of a sequence is $n^2 + 4n + 4$.

(a) Work out the 10th term.

(b) Is 400 a term of the sequence? You must show your working.

(a) $10^2 + 4 \times 10 + 4 = 144$

(b) Solve $n^2 + 4n + 4 = 400$, so $n^2 + 4n - 396 = 0$

Factorise or use the formula: $n = 18$ or $n = -22$

$n = 18$ is a positive whole number, so 400 is term number 18

Answer: (a) 144 (b) Yes, it is the 18th term

8 Exponential model y = ab^t

$y = ab^x$, where a and b are positive constants.

When $x = 1$, $y = 240$. When $x = 3$, $y = 345.6$.

Find the values of a and b, and the value of y when $x = 4$.

Divide: $b^2 = 345.6 \div 240 = 1.44$

$b = 1.2$

$a \times 1.2^1 = 240$ so $a = 200$

$y = 200 \times 1.2^4 = 414.72$

Answer: a = 200, b = 1.2, y = 414.72

9 Cone made from a sector

A sector of a circle has radius 15 cm and angle 120°.

The sector is made into a cone by joining the two straight edges together.

(a) Work out the radius of the base of the cone.

(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{120}{360} \times 2\pi \times 15 = 10\pi$ cm

This is the circumference of the base: $2\pi r = 10\pi$, so $r = 5$ cm

(b) Slant height = 15 cm, so height$^2 = 15^2 - 5^2 = 200$, height = $10\sqrt{2}$ cm

Volume = $\frac{1}{3} \times \pi \times 5^2 \times 10\sqrt{2} = \frac{250\sqrt{2}}{3}\pi$ cm^3

Answer: (a) 5 cm (b) $\frac{250\sqrt{2}}{3}\pi$ cm^3

10 Normal to y = kx^2: show that

P is the point $(p, \frac{3}{2}p^2)$ on the curve $y = \frac{3}{2}x^2$, where $p \neq 0$.

The normal to the curve at P meets the y-axis at the point Q.

Show that the y coordinate of Q is $\frac{3}{2}p^2 + \frac{1}{3}$.

$\frac{dy}{dx} = 3x$, so the gradient of the tangent at P is 3p

Normal gradient = $-\frac{1}{3p}$

Normal: $y - \frac{3}{2}p^2 = -\frac{1}{3p}(x - p)$

Put x = 0: $y = \frac{3}{2}p^2 + \frac{p}{3p} = \frac{3}{2}p^2 + \frac{1}{3}$

Answer: shown: $y = \frac{3}{2}p^2 + \frac{1}{3}$