Day 227 **IGCSE 10-a-day: Model Solutions**

1 Compound interest

Tariq invests £4,000. The rates of interest for the next 3 years are 4%, 5% and 6%.
Compound interest is paid. Work out the value of the investment after 3 years.
Give your answer to the nearest penny. (Calculator)

Multipliers: 1.04, 1.05 and 1.06
$4000 \times 1.04 \times 1.05 \times 1.06 = 4630.08$
To the nearest penny: £4,630.08
Answer: £4,630.08

2 Given a stationary point, find k

The curve $y = x^3 + kx$ has a stationary point at $x = -1$.
(a) Find the value of k.
(b) Find the coordinates of the other stationary point.

$\frac{dy}{dx} = 3x^2 + k$
$x = -1$: $3 + k = 0$, so $k = -3$
$\frac{dy}{dx} = 3x^2 - 3 = 3(x + 1)(x - 1)$
Other root: $x = 1$
$y = -2$
Answer: (a) k = −3 (b) (1, −2)

3 Where a line meets the axes

The line L has equation $12x - 5y = 120$.
L crosses the x-axis at the point A and the y-axis at the point B.
(a) Find the coordinates of A and B.
(b) Work out the area of triangle OAB, where O is the origin.

(a) At A, $y = 0$: $12x = 120$, so $x = 10$. A is (10, 0)
At B, $x = 0$: $-5y = 120$, so $y = -24$. B is (0, −24)
(b) OA = 10 and OB = 24, and angle AOB = 90°
Area $= \frac{1}{2} \times 10 \times 24 = 120$
Answer: (a) A(10, 0), B(0, −24) (b) 120

4 Weighted percentages

The ratio of juniors to seniors in a school is 1 : 2.
80% of the juniors and 35% of the seniors play a sport.
What percentage of all of these students play a sport?

Imagine 60 students in total: 20 juniors and 40 seniors
80% of 20 = 16 and 35% of 40 = 14
Together: 30 out of 60
$30 \div 60 \times 100 = 50\%$
Answer: 50%

5 Tangent perpendicular to a line

The tangent at the point P on the curve $y = 3x^2 + x + 7$ is perpendicular to the line $x - 5y = -3$.
Find the coordinates of P.

The line is $y = \frac{1}{5}x + \frac{3}{5}$, so its gradient is $\frac{1}{5}$
The tangent has gradient −5
$\frac{dy}{dx} = 6x + 1 = -5$ so $x = -1$
$y = 9$
Answer: (−1, 9)

6 Describe a transformation from a diagram

Triangle T is mapped onto triangle T′ by a single transformation.
(a) Describe fully this transformation.
(b) Write down the matrix that represents it.

(a) Compare corresponding vertices, e.g. (2, 1) goes to (−2, 1)
The transformation is a reflection in the y-axis

(b) The columns show where (1, 0) and (0, 1) go: $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

Answer: (a) Reflection in the y-axis (b) $\begin{pmatrix} \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

7 Cone made from a sector

A sector of a circle has radius 6 cm and angle 180°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{180}{360} \times 2\pi \times 6 = 6\pi$ cm

This is the circumference of the base: $2\pi r = 6\pi$, so r = 3 cm

(b) Slant height = 6 cm, so height$^2 = 6^2 - 3^2 = 27$, height = $3\sqrt{3}$ cm

Volume = $\frac{1}{3} \times \pi \times 3^2 \times 3\sqrt{3} = 9\sqrt{3}\pi$ cm^3

Answer: (a) 3 cm (b) $\mathbf{9\sqrt{3}\pi\ cm^3}$

8 Minimum point of y = ax + b/x

The diagram shows a sketch of the curve $y = x + \frac{9}{x}$ for x > 0.

The curve has a minimum point at M.
Find the coordinates of M, justifying that it is a minimum.

$y = x + 9x^{-1}$, so $\frac{dy}{dx} = 1 - 9x^{-2}$

$\frac{dy}{dx} = 0$: $1 = \frac{9}{x^2}$, so $x^2 = 9$ and x = 3 (x > 0)

$\frac{d^2y}{dx^2} = 18x^{-3} = \frac{2}{3} > 0$, so a minimum

$y = 3 + \frac{9}{3} = 6$

Answer: M (3, 6)

9 HCF and LCM

The HCF of two numbers is 8 and their LCM is 240.
One of the numbers is 40.
Work out the other number.

HCF × LCM = product of the two numbers
8 × 240 = 1920
Other number = 1920 ÷ 40 = 48
Answer: 48

10 Simultaneous equations with an inverse matrix

$M = \begin{pmatrix} -2 & 5 \\ -3 & 6 \end{pmatrix}$.

(a) Find M^{-1}.

(b) Use your answer to (a) to solve

$-2x + 5y = -11$

$-3x + 6y = -15$

(a) det M = 3 so $M^{-1} = \frac{1}{3}\begin{pmatrix} 6 & -5 \\ 3 & -2 \end{pmatrix}$

(b) The equations are $\begin{pmatrix} -2 & 5 \\ -3 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -11 \\ -15 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 6 & -5 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} -11 \\ -15 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

Answer: (a) $\mathbf{\frac{1}{3}\begin{pmatrix} 6 & -5 \\ 3 & -2 \end{pmatrix}}$ **(b) x = 3, y = −1**