Day 226 **IGCSE 10-a-day: Model Solutions**

1 Doubling and repeated growth

The number of a population of insects doubles every 5 days. At the start there are 200.
How many are there after 10 days?

10 ÷ 5 = 2 lots of 5 days
Number = 200×2^2
= 800
Answer: 800

2 Fraction problems

A tank is $\frac{3}{8}$ full.

72 more litres of water are added and the tank is then $\frac{3}{5}$ full.

Work out the capacity of the tank.

Fraction added = $\frac{3}{5} - \frac{3}{8} = \frac{9}{40}$

$\frac{9}{40}$ of the capacity is 72 litres

Capacity = $72 \div \frac{9}{40} = 320$

Answer: 320 litres

3 Vertical distance between stationary points

The curve $y = x^3 - 6x^2 + 9x - 8$ has two stationary points, A and B.
Work out the vertical distance between A and B.

$\frac{dy}{dx} = 3x^2 - 12x + 9 = 3(x - 1)(x - 3)$
$x = 1$ and $x = 3$
At $x = 1$, $y = -4$; at $x = 3$, $y = -8$
Difference = $-4 - (-8) = 4$
Answer: 4

4 Sample space and conditional probability

A fair 4-sided dice and a fair 6-sided dice are each thrown once. The first score and the second score are recorded.
(a) Work out the probability that the sum is greater than 6.
(b) Given that at least one number is 3, work out the probability that the sum is greater than 6.

(a) There are $4 \times 6 = 24$ equally likely outcomes, and 10 of them satisfy: the sum is greater than 6

Probability = $\frac{5}{12}$

(b) 9 outcomes satisfy: at least one number is 3. Of these, 4 also satisfy: the sum is greater than 6

Probability = $\frac{4}{9}$

Answer: (a) $\frac{5}{12}$ (b) $\frac{4}{9}$

5 Quadratic sequences

Here are the first five terms of a quadratic sequence.
3, 17, 39, 69, 107
Find an expression for the nth term.

First differences: 14, 22, 30, 38
Second differences: 8, 8, 8, so $a = 8 \div 2 = 4$
Subtract $4n^2$ from each term: $-1, 1, 3, 5, 7$
The remaining linear sequence has nth term $2n - 3$
nth term = $4n^2 + 2n - 3$
Answer: $4n^2 + 2n - 3$

6 Stationary points and their nature

$y = -x^3 + 3x^2$

(a) Find the coordinates of the stationary points.

(b) Use $\frac{d^2y}{dx^2}$ to decide whether each is a maximum or a minimum.

$\frac{dy}{dx} = -3x^2 + 6x = -3x(x - 2)$

$\frac{dy}{dx} = 0$ gives $x = 0$ or $x = 2$, so the points are (0, 0) and (2, 4)

$\frac{d^2y}{dx^2} = -6x + 6$

$x = 0$: $\frac{d^2y}{dx^2} = 6$, which is positive, so a minimum

$x = 2$: $\frac{d^2y}{dx^2} = -6$, which is negative, so a maximum

Answer: (a) (0, 0) and (2, 4) (b) (0, 0) is a minimum, (2, 4) is a maximum

7 Exact trigonometric ratios from a given ratio

x is an obtuse angle and $\sin x = \frac{1}{7}$.

(a) Find the exact value of cos x.

(b) Find the exact value of tan x. Give each answer as a fraction or a surd. Do not use a calculator.

(a) $\cos^2 x = 1 - (\frac{1}{7})^2 = \frac{48}{49}$

x is obtuse, so cos x is negative: $\cos x = -\frac{4\sqrt{3}}{7}$

(b) $\tan x = \frac{\sin x}{\cos x} = \frac{1}{7} \div (-\frac{4\sqrt{3}}{7}) = -\frac{\sqrt{3}}{12}$

Answer: (a) $-\frac{4\sqrt{3}}{7}$ (b) $-\frac{\sqrt{3}}{12}$

8 Geometric sequences

The first three terms of a geometric sequence are

24, 72, 216

(a) Write down the common ratio.

(b) Work out the 7th term.

(a) $72 \div 24 = 3$

(b) nth term $= 24 \times 3^{n-1}$

7th term $= 24 \times 3^6 = 17496$

Answer: (a) 3 (b) 17496

9 Speed and a moving train

A train travels at a constant speed of 90 km/h.

It takes 30 seconds to go completely through a tunnel which is 590 m long.

Work out the length of the train, in metres.

Speed = 90 km/h = 90,000 ÷ 3600 = 25 m/s

Distance travelled in 30 s = 25 × 30 = 750 m

This is the length of the tunnel plus the length of the train

Train = 750 − 590 = 160 m

Answer: 160 m

10 Show particle never at rest

A particle moves in a straight line. After t seconds its displacement from a fixed point is s metres, where $s = 2t^3 - 24t^2 + 98t - 1$

(a) Show that the particle is never at rest.

(b) Find the least speed of the particle and the time it occurs.

$v = \frac{ds}{dt} = 6t^2 - 48t + 98 = 6(t - 4)^2 + 2$

A square is never negative, so $v \geq 2 > 0$ and P is never at rest

Least value of v occurs when t = 4, giving 2

Answer: (b) 2 m/s when t = 4 s