Day 225 **IGCSE 10-a-day: Model Solutions**

1 Functions that commute

f(x) = 3x − 2 and g(x) = kx + 3, where k is a constant.
fg(x) = gf(x) for all values of x.
Find the value of k.

fg(x) = 3(kx + 3) − 2 = 3kx + 7
gf(x) = k(3x − 2) + 3 = 3kx − 2k + 3
Equate the constant terms: 7 = −2k + 3
−2k = 4, so k = −2
Answer: k = −2

2 Expand then differentiate

y = (x + 6)(x + 1)
Find $\frac{dy}{dx}$.

Expand: $y = x^2 + 7x + 6$
$x^2 \to 2x$, $7x \to 7$
$\frac{dy}{dx} = 2x + 7$
Answer: $\mathbf{\frac{dy}{dx} = 2x + 7}$

3 Simplifying algebraic fractions

Simplify fully $\frac{5x^2 - 80}{x^2 + 3x - 28}$

Factorise the top: 5(x − 4)(x + 4)
Factorise the bottom: (x − 4)(x + 7)
Cancel the common factor (x − 4): $\frac{5(x + 4)}{x + 7}$
Answer: $\mathbf{\frac{5(x + 4)}{x + 7}}$

4 Error intervals

The width of a table is 518 cm, correct to 3 significant figures.
Write down the error interval for the width of a table, using w.

The value has been rounded to the nearest 1
Half of this is 0.5, so go 0.5 below and 0.5 above
518 − 0.5 = 517.5 and 518 + 0.5 = 518.5
517.5 ≤ w < 518.5
Answer: 517.5 ≤ w < 518.5

5 Venn diagram with three sets: reading

The Venn diagram shows information about 54 pupils.
A: play football B: play netball C: play rugby
(a) Find n(A ∪ C).
(b) Find n(A ∪ B ∪ C)′.

(a) n(A ∪ C) = 7 + 8 + 8 + 5 + 4 + 7 = 39
(b) n(A ∪ B ∪ C)′ = 5
Answer: (a) 39 (b) 5

6 Volume and surface area of composite solids

A solid cylinder of radius 9 cm and height 12 cm has a hemispherical hollow of the same radius cut out of the top.
Work out the volume of the solid that is left. Give your answer in terms of π.

Cylinder: $\pi \times 9^2 \times 12 = 972\pi$

Hemisphere: $\frac{2}{3} \times \pi \times 9^3 = 486\pi$

Volume left = $972\pi - 486\pi = 486\pi$ cm^3

Answer: 486π cm^3

7 Points with a given gradient on y = ax + b/x

A curve has equation $y = x + \frac{3}{x}$.

Find the coordinates of the points on the curve where the gradient is -2.

$\frac{dy}{dx} = 1 - \frac{3}{x^2}$

Set $\frac{dy}{dx} = -2$: $3 = \frac{3}{x^2}$

$x^2 = 1$, so $x = 1$ or $x = -1$

$y = 4$ when $x = 1$ and $y = -4$ when $x = -1$

Answer: (1, 4) and (−1, −4)

8 Exponential growth and decay

There are 200 cells in a sample. The number of cells is multiplied by 4 every day.
After how many days will there first be more than 100,000 cells?

Number after n days = 200×4^n

n = 4: 51,200

n = 5: 204,800

204,800 is the first value above 100,000

Answer: 5 days

9 Greatest speed from displacement

A particle moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where
$s = -t^3 + 6t^2 + 10t$ for $t \geq 0$
Find the greatest speed of the particle, and the value of t at which it occurs.

$v = \frac{ds}{dt} = -3t^2 + 12t + 10$

$a = \frac{dv}{dt} = -6t + 12$

Maximum v when a = 0: $-6t + 12 = 0$, so t = 2

$\frac{d^2v}{dt^2} = -6 < 0$, so v is a maximum

v = 22

Answer: 22 m/s when t = 2 s

10 Estimated mean with an unknown frequency

The table gives information about the masses of some eggs.
An estimate for the mean is 11.5.
Work out the value of x.

Mid-points: 2.5, 7.5, 12.5, 17.5, 22.5

Estimated mean = $\Sigma fm \div \Sigma f$, so $(560 + 17.5x) \div (56 + x) = 11.5$

$560 + 17.5x = 11.5(56 + x) = 644 + 11.5x$

$(17.5 - 11.5)x = 644 - 560$

$6x = 84$, so $x = 14$

Answer: x = 14