Day 224 **IGCSE 10-a-day: Model Solutions**

1 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 9^2 + 12^2 = 225$
$AG^2 = AC^2 + CG^2 = 225 + 8^2 = 289$
AG = 17 cm

(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{8}{\sqrt{225}}$

angle GAC = 28.1° (1 d.p.)
Answer: (a) 17 cm (b) 28.1°

2 Inequalities from a region

Region R is shaded. Solid boundaries are included, dashed ones are not.
(a) Write down the three inequalities that define R.
(b) How many points with integer coordinates lie in R?

The vertical line is x = 1, the horizontal line is y = 1
The sloping line passes through (7, 1) and (1, 4): x + 2y = 9
R is to the right of x = 1, above y = 1 and below the sloping line. Solid lines are included, dashed lines are not:
$x > 1$, $y \geq 1$, $x + 2y < 9$
Count the integer points column by column: 3 + 2 + 2 + 1 + 1 = 9
Answer: (a) $x > 1$, $y \geq 1$, $x + 2y < 9$ (b) 9

3 Solving quadratics by factorising

Solve $12x^2 - 31x = -7$

Rearrange so that one side is 0: $12x^2 - 31x + 7 = 0$
Factorise: $(3x - 7)(4x - 1) = 0$
$3x - 7 = 0$ or $4x - 1 = 0$
$x = \frac{7}{3}$ or $x = \frac{1}{4}$
Answer: $x = \frac{1}{4}$ or $x = \frac{7}{3}$

4 Where is the second derivative zero?

$y = 3x^3 + 18x^2 - 7x + 4$

(a) Find $\frac{d^2y}{dx^2}$.

(b) Find the value of x for which $\frac{d^2y}{dx^2} = 0$, and work out the gradient of the curve at this value of x.

$\frac{dy}{dx} = 9x^2 + 36x - 7$
$\frac{d^2y}{dx^2} = 18x + 36$
$\frac{d^2y}{dx^2} = 0$ gives $18x + 36 = 0$, so $x = -2$
$\frac{dy}{dx} = -43$ when $x = -2$
Answer: (a) $18x + 36$ (b) $x = -2$, gradient -43

5 Radius from chord and height of arc

A circular arc has a chord AB of length 18 cm.
The greatest distance between the chord and the arc is 1 cm.
Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so OM = $r - 1$

AM = $18 \div 2 = 9$. Pythagoras in triangle OMA: $r^2 = (r - 1)^2 + 9^2$

$r^2 = r^2 - 2r + 1 + 81$

$2r = 82$, so $r = 41$ cm

Answer: 41 cm

6 Increasing and decreasing with powers

$f(x) = x^{\frac{3}{2}} - 6x$ for $x > 0$
(a) Find the range of values of x for which f is a decreasing function.
(b) Find the least value of f(x).

$f'(x) = \frac{3}{2}x^{\frac{1}{2}} - 6$

$f'(x) < 0$ when $\frac{3}{2}x^{\frac{1}{2}} < 6$, so $x^{\frac{1}{2}} < 4$

Square both sides: $x < 16$, so $0 < x < 16$

f decreases then increases, so the least value is at $x = 16$: $f = 16^{\frac{3}{2}} - 6 \times 16 = 64 - 96 = -32$

Answer: (a) $0 < x < 16$ (b) -32

7 Area scale factor and determinant

Shape S has area 3 cm^2.
S is transformed by the matrix $\begin{pmatrix} k & 3 \\ 4 & 4 \end{pmatrix}$ and the image has area 12 cm^2.
Find the two possible values of k.

Area scale factor = $12 \div 3 = 4$
Determinant = $4k - 12 = \pm 4$
$4k - 12 = 4$ gives $k = 4$; $4k - 12 = -4$ gives $k = 2$
Answer: k = 2 or k = 4

8 Bounds of roots

The area of a square is 99 cm^2, correct to the nearest cm^2.
Work out the lower bound of the side length of the square.
Give your answer correct to 3 significant figures. (Calculator)

$98.5 \leq$ area < 99.5
Lower bound of side = $\sqrt{98.5}$
= 9.92472... = 9.92 (3 s.f.)
Answer: 9.92 cm

9 Venn diagrams with algebra

The Venn diagram shows how many of 23 students study French (F) or Spanish (S).
(a) Work out the value of x.
(b) A student is chosen at random from those who study Spanish. Work out the probability that this student also studies French.

(a) Total: $(x + 1) + x + (2x - 3) + 5 = 23$
$4x + 3 = 23$, so $x = 5$
(b) Students in the second circle: $x + 2x - 3 = 5 + 7 = 12$, of whom 5 are in both
Probability = $\frac{5}{12}$

Answer: (a) x = 5 (b) $\frac{5}{12}$

10 Wire cut in two

A piece of wire 84 cm long is cut into two pieces. One piece is bent to form a square of side x cm. The other piece is bent to form a rectangle whose length is three times its width.

(a) Show that the total area, A cm^2, enclosed by the two shapes is $A = x^2 + \frac{3(84 - 4x)^2}{64}$.

(b) Find the value of x for which A is least, and work out this least area.

Rectangle perimeter $84 - 4x = 8w$, so $w = \frac{84 - 4x}{8}$ and its area is $3w^2 = \frac{3(84 - 4x)^2}{64}$

$A = x^2 + \frac{3(84 - 4x)^2}{64} = \frac{7}{4}x^2 - \frac{63}{2}x + \frac{1323}{4}$

$\frac{dA}{dx} = \frac{7}{2}x - \frac{63}{2} = 0$, so $x = 9$. $\frac{d^2A}{dx^2} = \frac{7}{2} > 0$, so a minimum

Square: 81, rectangle: $18 \times 6 = 108$, so $A = 189$

Answer: x = 9, least area 189 cm^2