Day 223 **IGCSE 10-a-day: Model Solutions**

1 Gradient at a point

Work out the gradient of the curve $y = 2x^3 + 8x + 8$ at the point where $x = 1$.

$\frac{dy}{dx} = 6x^2 + 8$

When $x = 1$: gradient $= 14$

Answer: 14

2 Inequalities

(a) Solve $9 - 2x > 3$

(b) n is an integer. Write down all the values of n that satisfy

$-16 < 3n - 4 < -1$

(a) Subtract 9: $-2x > -6$

Divide by -2 and reverse the inequality sign: $x < 3$

(b) Add 4: $-12 < 3n < 3$

Divide by 3: $-4 < n < 1$

Integers: $-3, -2, -1, 0$

Answer: (a) $x < 3$ (b) $-3, -2, -1, 0$

3 Where is the second derivative zero?

$y = x^3 + 4x + 8$

(a) Find $\frac{d^2y}{dx^2}$.

(b) Find the value of x for which $\frac{d^2y}{dx^2} = 0$, and work out the gradient of the curve at this value of x.

$\frac{dy}{dx} = 3x^2 + 4$

$\frac{d^2y}{dx^2} = 6x$

$\frac{d^2y}{dx^2} = 0$ gives $6x = 0$, so $x = 0$

$\frac{dy}{dx} = 4$ when $x = 0$

Answer: (a) $6x$ (b) $x = 0$, gradient 4

4 Forming and solving quadratic equations

A rectangle has length $(3x + 5)$ cm and width $(x + 5)$ cm.

The area of the rectangle is 312 cm^2.

(a) Show that $3x^2 + 20x - 287 = 0$

(b) Work out the perimeter of the rectangle.

(a) $(3x + 5)(x + 5) = 312$, so $3x^2 + 20x + 25 = 312$

$3x^2 + 20x - 287 = 0$

(b) $3x^2 + 20x - 287 = 0$: $(x - 7)(3x + 41) = 0$

$x = 7$ ($x = -\frac{41}{3}$ is not possible as lengths are positive)

Length = 26 cm and width = 12 cm, so perimeter $= 2 \times (26 + 12) = 76$ cm

Answer: (b) 76 cm

5 Rational and irrational numbers

Here are six numbers.

$\sqrt[3]{27}, \sqrt{\frac{9}{16}}, (\sqrt{6}+2)(\sqrt{6}-2), \sqrt{49}, \sqrt{2}\times\sqrt{6}, \sqrt{50}$

Write down the irrational numbers.

Simplify each number:

$\sqrt[3]{27} = 3; \sqrt{\frac{9}{16}} = \frac{3}{4}; (\sqrt{6}+2)(\sqrt{6}-2) = 6-4 = 2$

$\sqrt{49} = 7; \sqrt{2}\times\sqrt{6} = \sqrt{12} = 2\sqrt{3}; \sqrt{50} = 5\sqrt{2}$

Answer: $\sqrt{2}\times\sqrt{6}, \sqrt{50}$

6 Cosine rule: finding an angle

Work out the size of angle B.

Give your answer correct to 1 decimal place. (Calculator)

$\cos B = \frac{4.5^2 + 7.6^2 - 9^2}{2\times 4.5\times 7.6}$

$\cos B = -0.0437$

$B = 92.5°$

Answer: 92.5°

7 Sine rule

The diagram shows triangle ABC. (Calculator)

Angle BAC = 43° and angle ABC is acute.

Work out the size of angle ABC, marked θ. Give your answer correct to 1 decimal place.

The diagram is not drawn accurately.

Use the sine rule: $\frac{\sin\theta}{AC} = \frac{\sin A}{BC}$

$\frac{\sin\theta}{5} = \frac{\sin 43°}{8}$

$\sin\theta = \frac{5\times\sin 43°}{8} = 0.42625$

$\theta = \sin^{-1}(0.42625) = 25.2°$ (1 d.p.)

Answer: $\theta = 25.2°$

8 Relative speed

Grace and Tariq live 330 km apart.

At 10:00 they set off towards each other on the same road.

Grace drives at 90 km/h and Tariq drives at 90 km/h.

(a) At what time do they meet?

(b) How far from Grace's home do they meet?

Closing speed = 90 + 90 = 180 km/h

(a) Time = 330 ÷ 180 = $\frac{11}{6}$ hours = 110 minutes, so they meet at 11:50

(b) Distance = $90\times\frac{11}{6}$ = 165 km

Answer: (a) 11:50 (b) 165 km

9 Normal to y = kx^2: show that

P is the point $(t, \frac{9}{2}t^2)$ on the curve $y = \frac{9}{2}x^2$, where $t \neq 0$.

The normal to the curve at P meets the y-axis at the point Q.

Show that the y coordinate of Q is $\frac{9}{2}t^2 + \frac{1}{9}$.

$\frac{dy}{dx} = 9x$, so the gradient of the tangent at P is 9t

Normal gradient $= -\frac{1}{9t}$

Normal: $y - \frac{9}{2}t^2 = -\frac{1}{9t}(x - t)$

Put x = 0: $y = \frac{9}{2}t^2 + \frac{t}{9t} = \frac{9}{2}t^2 + \frac{1}{9}$

Answer: shown: $y = \frac{9}{2}t^2 + \frac{1}{9}$

10 Probability and quadratic equations

A bag contains x white discs and 4 green discs.
Two discs are taken at random without replacement.

The probability that both are white is $\frac{6}{13}$.

(a) Show that $7x^2 - 55x - 72 = 0$.
(b) Hence work out the number of discs in the bag.

(a) P(both white) $= \frac{x}{x+4} \times \frac{x-1}{x+3} = \frac{6}{13}$

$13\,x(x - 1) = 6(x + 4)(x + 3)$

$13x^2 - 13x = 6x^2 + 42x + 72$ so $7x^2 - 55x - 72 = 0$

(b) Solve: $x = 9$ or $x = -\frac{8}{7}$

x cannot be negative, so x = 9

Number of discs = 9 + 4 = 13

Answer: x = 9, so 13 discs in the bag