Day 222 **IGCSE 10-a-day: Model Solutions**

1 Laws of indices

Given that $\frac{5^3 \times 25}{25^2} = 5^n$,

find the value of n.

$25 = 5^2$

Numerator: $5^3 \times 5^2 = 5^5$

Denominator: 5^4

Subtract the powers: $5 - 4 = 1$

Answer: n = 1

2 Expand then differentiate

$y = x(x + 1)(2x - 2)$

Find $\frac{dy}{dx}$.

Expand: $y = 2x^3 - 2x$

$2x^3 \rightarrow 6x^2$, $-2x \rightarrow -2$

$\frac{dy}{dx} = 6x^2 - 2$

Answer: $\frac{dy}{dx} = 6x^2 - 2$

3 Substitution

Work out the value of $3m - n^3$
when $m = -4$, $n = -4$ and $k = 2$.

Substitute the values: $3 \times (-4) - (-4)^3$

$= -12 - (-64)$

$= 52$

Answer: 52

4 Venn diagram: shaded region notation

Write down the set notation for the region shaded in the Venn diagram.

Identify which regions are shaded, using the circles X and Y

The shaded region is $X \cap Y$

Answer: $X \cap Y$

5 Speed-time graphs

The graph shows the speed of a car during 18 seconds.
(a) Work out the acceleration during the first 7 seconds.
(b) Work out the average speed over the whole 18 seconds.

(a) Acceleration = change in speed ÷ time = $(14 - 0) \div 7 = 2$ m/s^2

(b) Distance = area under the graph

Trapezium: $\frac{0 + 14}{2} \times 7 = 49$; rectangle: $14 \times 9 = 126$; triangle: $\frac{1}{2} \times 2 \times 14 = 14$

Total = 189 m, so average speed = $189 \div 18 = 10.5$ m/s

Answer: (a) 2 m/s^2 (b) 10.5 m/s

6 Transformations of graphs

The graph of $y = f(x)$ has a maximum point at $(0, -4)$.
Write down the coordinates of the turning point of the graph of
(a) $y = 2f(x)$
(b) $y = f(x - 1) + 1$

(a) Start with the turning point $(0, -4)$
Apply the transformation to the point: $(0, -8)$
(b) $f(x - 1)$ changes the x-coordinate to 1, and $f(x) + 1$ changes the y-coordinate to -3
Turning point: $(1, -3)$
Answer: (a) $(0, -8)$ (b) $(1, -3)$

7 Greatest and least values on an interval

$f(x) = -x^3 - 3x^2 - 5$ for $-4 \leq x \leq 2$.
Find the greatest value and the least value of $f(x)$ on this interval.

$f'(x) = -3x^2 - 6x = 0$ gives $x = -2$ or $x = 0$ (both in the interval)
$f(-4) = 11$, $f(-2) = -9$, $f(0) = -5$, $f(2) = -25$
Compare the turning points and both end values
Answer: Greatest 11, least -25

8 Tree diagrams and reverse conditional probability

Machine A makes 60% of the items and machine B makes the rest.
9% of items from A are faulty and 2% of items from B are faulty.
An item is chosen at random.
(a) Work out the probability that the item is faulty.
(b) The item is faulty. Work out the probability that it came from machine A.

(a) $0.6 \times 0.09 = 0.054$ and $0.4 \times 0.02 = 0.008$
P(faulty) $= 0.054 + 0.008 = 0.062$
(b) $\frac{0.054}{0.062} = \frac{27}{31}$

Answer: (a) 0.062 (b) $\frac{27}{31}$

9 Algebraic proof (harder)

Show algebraically that the sum of the squares of any two odd numbers is 2 more than a multiple of 8.

Let the odd numbers be $2a + 1$ and $2b + 1$.
$(2a + 1)^2 + (2b + 1)^2 = 4a^2 + 4a + 4b^2 + 4b + 2 = 4[a(a + 1) + b(b + 1)] + 2$
$a(a + 1)$ and $b(b + 1)$ are each a product of consecutive integers, so each is even
So $a(a + 1) + b(b + 1)$ is even, and $4 \times$ an even number is a multiple of 8
Hence the sum is 2 more than a multiple of 8
Answer: Proved

10 Find constants from a gradient

A curve has equation $y = ax^2 + \frac{b}{x}$, where a and b are constants.

The point $(1, -3)$ lies on the curve and the gradient of the curve at this point is -3.
Find the values of a and b.

$\frac{dy}{dx} = 2ax - bx^{-2} = 2ax - \frac{b}{x^2}$
Point on curve: $a + b = -3$. Gradient: $2a - b = -3$
Solving simultaneously gives $a = -2$ and $b = -1$
Answer: $a = -2$, $b = -1$