Day 221 **IGCSE 10-a-day: Model Solutions**

1 Tests of divisibility

The 4-digit number 55k2 is a multiple of 9.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 9 if its digit sum is a multiple of 9
Digit sum = 5 + 5 + 2 + k = 12 + k
This must be a multiple of 9, so k = 6
Answer: k = 6

2 Expand then differentiate

$y = (x - 3)(x + 3)$
Find $\frac{dy}{dx}$.

Expand: $y = x^2 - 9$
$x^2 \rightarrow 2x$
$\frac{dy}{dx} = 2x$
Answer: $\mathbf{\frac{dy}{dx} = 2x}$

3 Completing the square

(a) Write $x^2 - 8x + 12$ in the form $(x + p)^2 + q$
(b) Write down the coordinates of the turning point of the curve $y = x^2 - 8x + 12$

(a) Half of -8 is -4, so $x^2 - 8x = (x - 4)^2 - 16$
$x^2 - 8x + 12 = (x - 4)^2 - 16 + 12$
$= (x - 4)^2 - 4$
(b) The minimum is when $x - 4 = 0$, so $x = 4$ and $y = -4$
Answer: (a) $(x - 4)^2 - 4$ (b) $(4, -4)$

4 Mixtures and percentage profit

A shop mixes two types of coffee.
Type A costs £8.00 per kg and type B costs £5.00 per kg.
They are mixed in the ratio A : B = 5 : 1.
The mixture is sold for £9.75 per kg.
Work out the percentage profit.

Cost of 6 kg of mixture = 5 × £8.00 + 1 × £5.00 = £45.00
Cost per kg = £45.00 ÷ 6 = £7.50
Profit per kg = £9.75 − £7.50 = £2.25
Percentage profit = 2.25 ÷ 7.50 × 100 = 30%
Answer: 30%

5 Cosine rule and area of a triangle

The diagram shows triangle ABC. (Calculator)
(a) Work out angle ACB, marked θ, to 1 decimal place.
(b) Work out the area of triangle ABC to 3 significant figures.
The diagram is not drawn accurately.

(a) $\cos\theta = \frac{8^2 + 11^2 - 6^2}{2 \times 8 \times 11} = \frac{149}{176}$
$\theta = 32.2°$ (1 d.p.)
(b) Area $= \frac{1}{2} \times 8 \times 11 \times \sin 32.2°$
$= 23.4\ \text{cm}^2$ (3 s.f.)
Answer: (a) 32.2° (b) 23.4 cm²

6 Tree diagrams without replacement

A bag contains 4 orange beads, 4 purple beads and 1 pink bead.
Two beads are taken at random without replacement.
Work out the probability that (a) at least one is orange, (b) exactly one is orange.

(a) P(at least one orange) = $1 - \text{P(none)} = 1 - \frac{5}{9}\times\frac{4}{8} = \frac{13}{18}$

(b) P(exactly one orange) = $\frac{4}{9}\times\frac{5}{8} + \frac{5}{9}\times\frac{4}{8} = \frac{5}{9}$

Answer: (a) $\frac{13}{18}$ (b) $\frac{5}{9}$

7 Fastest rate of growth

The number of fish in a lake, N, after t months is modelled by
$N = -t^3 + 15t^2 + 7t + 1000$ for $0 \leq t \leq 20$
Find the time at which N is increasing most rapidly, and the rate of increase at this time.

$\frac{dN}{dt} = -3t^2 + 30t + 7$

Rate is greatest when $\frac{d^2N}{dt^2} = -6t + 30 = 0$, so $t = 5$

$\frac{d^2N}{dt^2}$ is decreasing (gradient -6), so the rate is a maximum

$\frac{dN}{dt} = 82$ when $t = 5$

Answer: t = 5, rate 82 per month

8 Three sets from information

78 pupils were asked which activities they do.
A: play football B: play netball C: play rugby
$n(A) = 52$, $n(B) = 46$, $n(C) = 38$, $n(A \cap B) = 28$, $n(A \cap C) = 24$, $n(B \cap C) = 22$, $n(A \cap B \cap C) = 14$.
(a) Work out how many are in none of A, B and C.
(b) Work out how many are in exactly one of A, B and C.

$n(A \cup B \cup C) = 52 + 46 + 38 - 28 - 24 - 22 + 14 = 76$
(a) Number in none of the sets $= 78 - 76 = 2$
(b) Only A $= 52 - 28 - 24 + 14 = 14$
Only B = 10, only C = 6, so exactly one $= 14 + 10 + 6 = 30$
Answer: (a) 2 (b) 30

9 Curves that touch

The curves $y = -x^2 + 5x - 2$ and $y = -3x^2 + x - 4$ touch at one point.
Find the coordinates of this point and the equation of the common tangent.

Equate the two expressions: $2x^2 + 4x + 2 = 0$
That is $2(x + 1)^2 = 0$ with a repeated root $x = -1$
$y = -8$, so the point is $(-1, -8)$
$\frac{dy}{dx} = -2x + 5$ on the first curve: gradient 7
Tangent: $y = 7x - 1$
Answer: $(-1, -8)$, $y = 7x - 1$

10 Bearings with sine and cosine rules

A ship sails 15 km from P to Q on a bearing of 050°.
It then sails 6 km from Q to R on a bearing of 151°.
(a) Work out PR (3 s.f.). (b) Work out the bearing of R from P (nearest degree). (Calculator)

Angle PQR = 050° + 180° − 151° = 79°

$PR^2 = 15^2 + 6^2 - 2 \times 15 \times 6 \times \cos 79°$ so PR = 15.1 km

$\cos QPR = \frac{15^2 + 15.055^2 - 6^2}{2 \times 15 \times 15.055}$, angle QPR = 23.03°

Bearing of R from P = 050° + 23.03° = 073°

Answer: (a) 15.1 km (b) 073°