Day 220 **IGCSE 10-a-day: Model Solutions**

1 Inverse proportion

y is inversely proportional to the square root of x.
When x = 36, y = 3.
Find the positive value of x when y = 18.

$y = \frac{k}{\sqrt{x}}$

$3 = \frac{k}{6}$, so $k = 3 \times 6 = 18$

$18 = \frac{18}{\sqrt{x}}$, so $\sqrt{x} = \frac{18}{18} = 1$

$x = 1^2 = 1$

Answer: x = 1

2 Cosine rule: finding a side

Work out the length of BC.
Give your answer correct to 3 significant figures. (Calculator)

$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos A$

$BC^2 = 10.9^2 + 14.4^2 - 2 \times 10.9 \times 14.4 \times \cos 137°$

$BC^2 = 555.76$

$BC = 23.6$

Answer: 23.6 cm

3 Point with a given gradient (quadratic)

A curve has equation $y = -2x^2 + 6x - 9$.
The gradient at the point P on the curve is -10.
Find the coordinates of P.

$\frac{dy}{dx} = -4x + 6$

$-4x + 6 = -10$, so $-4x = -16$

$x = 4$

$y = -17$

Answer: (4, −17)

4 Laws of indices with algebra

Simplify fully $\frac{9x^4y \times 7x^3y^5}{3xy^4}$

Give your answer in the form kx^ay^b

Numbers: $9 \times 7 \div 3 = 21$

Powers of x: $4 + 3 - 1 = 6$

Powers of y: $1 + 5 - 4 = 2$

$= 21x^6y^2$

Answer: $21x^6y^2$

5 Kinematics with roots

A particle moves in a straight line. After t seconds its displacement from O is s metres, where

$s = 16t^{\frac{3}{2}} - 2t^2$ $(t \geq 0)$

Find the velocity and the acceleration of the particle when t = 9.

$v = \frac{ds}{dt} = -4t + 24t^{\frac{1}{2}}$

$a = \frac{dv}{dt} = -4 + 12t^{-\frac{1}{2}}$

When t = 9: v = 36 and a = 0

Answer: v = 36 m/s, a = 0 m/s^2

6 Composite with an inverse function

$f(x) = 4x - 4$ and $g(x) = 3x^2 - 2x - 4$.
(a) Find $f^{-1}g(x)$.
(b) Work out $f^{-1}g(-2)$.

(a) $f^{-1}(x) = \frac{x + 4}{4}$

$f^{-1}g(x) = \frac{3x^2 - 2x - 4 + 4}{4} = \frac{3x^2 - 2x}{4}$

(b) $g(-2) = 12$, so $f^{-1}g(-2) = \frac{12 + 4}{4}$

Answer: (a) $\frac{3x^2 - 2x}{4}$ (b) 4

7 Surds and fractions

Simplify $\frac{1}{7 + \sqrt{7}} + \frac{1}{7 - \sqrt{7}}$
Give your answer with a rational denominator.

Common denominator: $(7 + \sqrt{7})(7 - \sqrt{7}) = 49 - 7 = 42$
Numerator: $(7 - \sqrt{7}) + (7 + \sqrt{7})$
$= 14$
$\frac{14}{42} = \frac{1}{3}$

Answer: $\frac{1}{3}$

8 Number of solutions of f(x) = k

The diagram shows a sketch of the curve $y = -x^3 + 3x^2 + 9x - 8$ and the line $y = k$.
The line meets the curve at three different points.
Find the range of possible values of k.

$\frac{dy}{dx} = -3x^2 + 6x + 9 = 0$ gives $x = -1$ and $x = 3$
$y = -13$ at $x = -1$ and $y = 19$ at $x = 3$
The line crosses three times when it lies between the two turning-point values
$-13 < k < 19$
Answer: $-13 < k < 19$

9 Bounds in geometry

The radius of a circle is 14 cm, correct to the nearest centimetre.
Work out the upper bound of the area of the circle. Give your answer in terms of π.

The radius is between $\frac{27}{2}$ cm and $\frac{29}{2}$ cm

For the upper bound, use $r = \frac{29}{2}$

Area $= \pi \times (\frac{29}{2})^2 = \frac{841}{4}\pi$ cm^2

Answer: $\frac{841}{4}\pi$ cm^2

10 Compound interest

Mo invests £2,500.
The account pays compound interest of 6% a year for the first 2 years,
then 3.5% a year for the next 2 years.
Work out the total amount after 4 years, to the nearest penny. (Calculator)

Multipliers: 1.06 for 2 years, then 1.035 for 2 years
£2,500 × 1.06^2 × 1.035^2
= £3,009.07
Answer: £3,009.07