Day 22 **IGCSE 10-a-day: Model Solutions**

1 Show particle never at rest

A particle moves in a straight line. After t seconds its displacement from a fixed point is s metres, where

$s = t^3 - 12t^2 + 52t - 1$

(a) Show that the particle is never at rest.

(b) Find the least speed of the particle and the time it occurs.

$v = \frac{ds}{dt} = 3t^2 - 24t + 52 = 3(t - 4)^2 + 4$

A square is never negative, so $v \geq 4 > 0$ and P is never at rest

Least value of v occurs when t = 4, giving 4

Answer: (b) 4 m/s when t = 4 s

2 Time zones

A flight leaves London at 08:10 (London time).

It takes 5 hours 20 minutes.

When it is 12:00 in London it is 04:00 in Los Angeles.

At what time, in Los Angeles time, does the flight land? Say if it is on a different day.

Arrival in London time = 08:10 + 5 h 20 min = 13:30

Los Angeles is 8 hours behind London, so subtract 8 hours

05:30

Answer: 05:30 the same day

3 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 36°.

(a) Work out the size of angle ACB.

(b) Work out the size of angle COB.

The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°

(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 36°

Angle COB is an exterior angle of triangle OAC: 36 + 36 = 72°

(or: the angle at the centre is twice the angle at the circumference)

Answer: (a) 90° (b) 72°

4 Changing the subject

Make x the subject of $y = 3x^3 + 15$

Subtract 15: $y - 15 = 3x^3$

Divide by 3: $x^3 = \frac{y - 15}{3}$

Cube root: $x = \sqrt[3]{\frac{y - 15}{3}}$

Answer: $\mathbf{x = \sqrt[3]{\frac{y - 15}{3}}}$

5 Inverse of a rational function

$f(x) = \frac{-x - 5}{3x + 2}$, $x \neq -\frac{2}{3}$.

Find $f^{-1}(x)$.

$y = \frac{-x - 5}{3x + 2}$, so $y(3x + 2) = -x - 5$

$3xy + 2y = -x - 5$

$3xy + x = -5 - 2y$

$x(3y + 1) = -5 - 2y$, so $x = \frac{-5 - 2y}{3y + 1}$

$f^{-1}(x) = \frac{-5 - 2x}{3x + 1}$

Answer: $\mathbf{f^{-1}(x) = \frac{-5 - 2x}{3x + 1}}$

6 Successive percentage change

The population of a town is increased by 60%.
The new amount is then decreased by 25%.
Work out the overall percentage change.

Multipliers: 1.6 and 0.75
$1.6 \times 0.75 = 1.2$
1.2 is an increase of 20%
Answer: 20% increase

7 Trapezium with vectors

OABC is a trapezium. OA is parallel to CB and CB = $\frac{2}{3}$OA.

$\mathbf{OA} = \mathbf{a}$ and $\mathbf{OC} = \mathbf{c}$.
(a) Find $\mathbf{AB}$ in terms of $\mathbf{a}$ and $\mathbf{c}$.
(b) AM : MB = 2 : 1. Find $\mathbf{OM}$.

(a) $\mathbf{CB} = \frac{2}{3}\mathbf{a}$

$\mathbf{OB} = \mathbf{OC} + \mathbf{CB} = \frac{2}{3}\mathbf{a} + \mathbf{c}$

$\mathbf{AB} = -\mathbf{OA} + \mathbf{OB} = -\frac{1}{3}\mathbf{a} + \mathbf{c}$

(b) $\mathbf{OM} = \mathbf{OA} + \frac{2}{3}\mathbf{AB} = \mathbf{a} + \frac{2}{3}(-\frac{1}{3}\mathbf{a} + \mathbf{c})$

$= \frac{7}{9}\mathbf{a} + \frac{2}{3}\mathbf{c}$

Answer: (a) $-\frac{1}{3}\mathbf{a} + \mathbf{c}$ (b) $\frac{7}{9}\mathbf{a} + \frac{2}{3}\mathbf{c}$

8 Greatest speed from displacement

A particle moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where
$s = -2t^3 + 30t^2 + 9t$ for $t \geq 0$
Find the greatest speed of the particle, and the value of t at which it occurs.

$v = \frac{ds}{dt} = -6t^2 + 60t + 9$

$a = \frac{dv}{dt} = -12t + 60$

Maximum v when a = 0: $-12t + 60 = 0$, so t = 5

$\frac{d^2v}{dt^2} = -12 < 0$, so v is a maximum

v = 159

Answer: 159 m/s when t = 5 s

9 Iteration: rearranging equations

(a) Show that the equation $x^3 - 2x - 23 = 0$ has a solution between x = 3 and x = 4.
(b) Show that the equation can be rearranged to give $x = \sqrt[3]{2x + 23}$

(a) Let $f(x) = x^3 - 2x - 23$. $f(3) = 3^3 - 2 \times 3 - 23 = -2$ and $f(4) = 4^3 - 2 \times 4 - 23 = 33$
f(3) is negative and f(4) is positive. There is a change of sign, so there is a solution between 3 and 4
(b) Rearrange $x^3 - 2x - 23 = 0$: $x^3 = 2x + 23$
Take the cube root of both sides: $x = \sqrt[3]{2x + 23}$
Answer: Shown

10 Points with a given gradient on y = ax + b/x

A curve has equation $y = -x - \frac{2}{x}$.

Find the coordinates of the points on the curve where the gradient is 1.

$\frac{dy}{dx} = -1 + \frac{2}{x^2}$

Set $\frac{dy}{dx} = 1$: $-2 = \frac{-2}{x^2}$

$x^2 = 1$, so $x = 1$ or $x = -1$

$y = -3$ when $x = 1$ and $y = 3$ when $x = -1$

Answer: (1, −3) and (−1, 3)