Day 219 **IGCSE 10-a-day: Model Solutions**

1 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 38°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 38°
Angle COB is an exterior angle of triangle OAC: 38 + 38 = 76°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 76°

2 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where
$s = -2t^3 + 6t^2 - 11t$ $(t > 0)$
Find (a) the velocity of P, (b) the acceleration of P, when t = 2.

$v = \frac{ds}{dt} = -6t^2 + 12t - 11$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = -12t + 12$

When t = 2: $v = -11$ and $a = -12$
Answer: (a) -11 m/s (b) -12 m/s^2

3 Matrix of a transformation

Write down the matrix that represents each transformation.
(a) Reflection in the line y = x
(b) Rotation of 90° clockwise about the origin

(a) Find where (1, 0) and (0, 1) go: they are the columns of the matrix

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Answer: (a) $\begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}$ **(b)** $\begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0} \end{pmatrix}$

4 Where is the second derivative zero?

$y = 2x^3 + 6x^2 + 9x - 6$

(a) Find $\frac{d^2y}{dx^2}$.

(b) Find the value of x for which $\frac{d^2y}{dx^2} = 0$, and work out the gradient of the curve at this value of x.

$\frac{dy}{dx} = 6x^2 + 12x + 9$

$\frac{d^2y}{dx^2} = 12x + 12$

$\frac{d^2y}{dx^2} = 0$ gives $12x + 12 = 0$, so $x = -1$

$\frac{dy}{dx} = 3$ when $x = -1$

Answer: (a) 12x + 12 (b) x = -1, gradient 3

5 Exact lengths and areas

A right-angled triangle has base $(3 + \sqrt{7})$ cm and height $(3 - \sqrt{7})$ cm.
Show that the area of the triangle is a rational number.
Work out the area.

Area $= \frac{1}{2} \times (3 + \sqrt{7})(3 - \sqrt{7})$

$(3 + \sqrt{7})(3 - \sqrt{7}) = 9 - 7 = 2$

Area $= \frac{1}{2} \times 2 = 1$

Answer: 1 cm^2

6 Extreme value of an expression in a region

x and y satisfy these inequalities:
$x \geq 3$, $y \leq 2$, $3x - 4y \leq 13$
Find the greatest possible value of $-3x + 3y$.

The region is a triangle. The extreme value occurs at a vertex.
Vertices: (3, 2), (7, 2), (3, −1)
Values of $-3x + 3y$: −3, −15, −12
Greatest value = −3
Answer: −3

7 Speed-time graphs

The graph shows the speed of a train during 18 seconds.
(a) Work out the acceleration during the first 3 seconds.
(b) Work out the average speed over the whole 18 seconds.

(a) Acceleration = change in speed ÷ time = $(24 - 6) \div 3 = 6$ m/s^2
(b) Distance = area under the graph
Trapezium: $\frac{6 + 24}{2} \times 3 = 45$; rectangle: $24 \times 9 = 216$; triangle: $\frac{1}{2} \times 6 \times 24 = 72$
Total = 333 m, so average speed = $333 \div 18 = 18.5$ m/s
Answer: (a) 6 m/s^2 (b) 18.5 m/s

8 Line and curve intersections

Solve the simultaneous equations
$y = x^2 + 3x - 1$
$y = 2x + 11$

Substitute: $x^2 + 3x - 1 = 2x + 11$
Rearrange: $x^2 + x - 12 = 0$
Factorise: $(x + 4)(x - 3) = 0$, so $x = -4$ or $x = 3$
When $x = -4$: $y = 2 \times (-4) + 11 = 3$
When $x = 3$: $y = 2 \times 3 + 11 = 17$
Answer: $x = -4$, $y = 3$ and $x = 3$, $y = 17$

9 Greatest speed from displacement

A particle moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where $s = -2t^3 + 30t^2 + 3t$ for $t \geq 0$

Find the greatest speed of the particle, and the value of t at which it occurs.

$v = \frac{ds}{dt} = -6t^2 + 60t + 3$

$a = \frac{dv}{dt} = -12t + 60$

Maximum v when a = 0: $-12t + 60 = 0$, so $t = 5$

$\frac{d^2v}{dt^2} = -12 < 0$, so v is a maximum

$v = 153$

Answer: 153 m/s when t = 5 s

10 Nested surds

Show that $\sqrt{22 + 8\sqrt{6}} = 4 + \sqrt{6}$

Square the right-hand side: $(4 + \sqrt{6})^2 = 16 + 8\sqrt{6} + 6$

$= 22 + 8\sqrt{6}$

So $\sqrt{22 + 8\sqrt{6}} = 4 + \sqrt{6}$

Answer: $\sqrt{22 + 8\sqrt{6}} = 4 + \sqrt{6}$