Day 218 **IGCSE 10-a-day: Model Solutions**

1 Differentiating polynomials

$y = -6x^4 + 3x^3 + 6x^2 + 6$

Find $\frac{dy}{dx}$

Differentiate each term: $x^n \to nx^{n-1}$ and a constant $\to 0$

$-6x^4 \to -24x^3$, $3x^3 \to 9x^2$, $6x^2 \to 12x$

$\frac{dy}{dx} = -24x^3 + 9x^2 + 12x$

Answer: $\mathbf{\frac{dy}{dx} = -24x^3 + 9x^2 + 12x}$

2 Bounds: finding an unknown length

A rectangle has area 15 cm^2, correct to 2 significant figures.
Its length is 4 cm, correct to 2 significant figures.
Work out the upper bound of the width of the rectangle. Give your answer correct to 3 significant figures. (Calculator)

Width = area ÷ length
Area: $14.5 \leq A < 15.5$. Length: $3.95 \leq l < 4.05$
UB width = UB area ÷ LB length = 15.5 ÷ 3.95 = 3.92
Answer: 3.92 cm

3 Tangent where the curve crosses the y-axis

The curve $y = x^3 - 3x^2 - 2x - 4$ crosses the y-axis at the point A.
The tangent to the curve at A meets the x-axis at B.
Find the coordinates of B.

At A, $x = 0$ and $y = -4$
$\frac{dy}{dx} = 3x^2 - 6x - 2$, so the gradient at A is -2
Tangent: $y = -2x - 4$
At B, $y = 0$: $-2x - 4 = 0$, so $x = -2$
Answer: (−2, 0)

4 Percentage change in area and volume

The length of a rectangle is decreased by 25%.
The width of the rectangle is increased by 20%.
Work out the percentage change in the area of the rectangle.

Length multiplier 0.75, width multiplier 1.2
Area multiplier = 0.75 × 1.2 = 0.9
0.9 − 1 = −0.1, so a 10% decrease
Answer: 10% decrease

5 Compound interest

Hassan invests £10,000 at 5% per year compound interest.
After how many whole years will the investment first be worth more than £11,900?
(Calculator)

Multiplier = 1.05
After 3 years: $10000 \times 1.05^3 = 11576.25$
After 4 years: $10000 \times 1.05^4 = 12155.06$
12155.06 is the first value above £11,900
Answer: 4 years

6 Inverse functions

$f(x) = \frac{-5x + 5}{3x + 2}$

(a) Find $f^{-1}(x)$.

(b) Find the value of $f^{-1}(-10)$.

(a) Let $y = \frac{-5x + 5}{3x + 2}$. Then $y(3x + 2) = -5x + 5$

$3xy + 2y = -5x + 5$

Collect x terms: $3xy + 5x = 5 - 2y$

Factorise: $x(3y + 5) = -2y + 5$, so $x = \frac{-2y + 5}{3y + 5}$

$f^{-1}(x) = \frac{-2x + 5}{3x + 5}$

(b) $f^{-1}(-10) = \frac{(-2) \times (-10) + 5}{3 \times (-10) - (-5)} = \frac{25}{-25} = -1$

Answer: (a) $f^{-1}(x) = \frac{-2x + 5}{3x + 5}$ (b) -1

7 Surds and fractions

Simplify $\frac{2}{2 + \sqrt{3}} - \frac{1}{2 - \sqrt{3}}$

Give your answer with a rational denominator.

Common denominator: $(2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 3 = 1$

Numerator: $2(2 - \sqrt{3}) - (2 + \sqrt{3})$

$= 2 - 3\sqrt{3}$

$\frac{2 - 3\sqrt{3}}{1} = 2 - 3\sqrt{3}$

Answer: $2 - 3\sqrt{3}$

8 Loci and regions

ABCD is a rectangle with AB = 13 cm and AD = 15 cm.
A point P inside the rectangle is such that AP ≤ 12 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 12 cm: P is inside the quarter circle of radius 12 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves

Area $= \frac{1}{2} \times \frac{1}{4} \times \pi \times 12^2 = 18\pi$ cm^2

Answer: 18π cm^2

9 Angles of elevation from two points

Points A and B are on level ground, 101 m apart, with B between A and the foot of a lighthouse.
The angle of elevation of the top of the lighthouse is 26° from A and 50° from B.
Work out the height of the lighthouse. Give your answer to 3 significant figures. (Calculator)

Let the top be T. Angle ATB = 50° − 26° = 24° (exterior angle of the triangle)

Sine rule in triangle ABT: $\frac{AT}{\sin 130°} = \frac{101}{\sin 24°}$

$AT = \frac{101 \times \sin 50°}{\sin 24°} = 190.22$

Height = AT × sin 26° = 83.4 m

Answer: 83.4 m

10 Open box from a square sheet

Squares of side x cm are cut from the corners of a 36 cm square cardboard sheet, which is folded into an open box of volume V cm^3.

(a) Show that $V = 4x^3 - 144x^2 + 1296x$.

(b) Find x for the maximum volume, and this volume.

Base is a square of side $36 - 2x$, so $V = x(36 - 2x)^2 = x(1296 - 144x + 4x^2) = 4x^3 - 144x^2 + 1296x$

$\frac{dV}{dx} = 12x^2 - 288x + 1296 = 0$, so $x^2 - 24x + 108 = 0$

$12(x - 6)(x - 18) = 0$, so $x = 6$ or $x = 18$

$x = 18$ is rejected (18 is more than half of 36). $\frac{d^2V}{dx^2} = 24x - 288$, which is -144 when $x = 6$, so a maximum

$V = 3456$

Answer: x = 6, V = 3456 cm^3