Day 217 **IGCSE 10-a-day: Model Solutions**

1 Rearranging when the subject appears twice

Make u the subject of $v = \frac{3u + 5}{u - 2}$

Multiply by $u - 2$: $v(u - 2) = 3u + 5$
Expand: $uv - 2v = 3u + 5$
Collect the u terms: $uv - 3u = 2v + 5$
Factorise: $u(v - 3) = 2v + 5$
$u = \frac{2v + 5}{v - 3}$

Answer: $u = \frac{2v + 5}{v - 3}$

2 f′(x) notation

$f(x) = -3x^4 + 5x^2 - 3$
(a) Find f′(x).
(b) Work out the value of f′(1).

(a) $-3x^4 \rightarrow -12x^3$, $5x^2 \rightarrow 10x$
$f'(x) = -12x^3 + 10x$
(b) $f'(1) = -2$
Answer: (a) $f'(x) = -12x^3 + 10x$ (b) -2

3 Writing inequalities from a situation

On a coach there are x adults and y children.
There are at most 60 people. There are at least 2 children for every adult. There are at least 4 adults.
(a) Write down three inequalities for x and y.
(b) Find the greatest number of children on the coach.

(a) At most 60 people: $x + y \leq 60$
At least 2 children for every adult: $y \geq 2x$
At least 4 adults: $x \geq 4$
(b) The most children occurs with the fewest adults: $x = 4$
$y \leq 60 - 4 = 56$ and $y \geq 8$, so $y = 56$ is allowed
Answer: (a) $x + y \leq 60$, $y \geq 2x$, $x \geq 4$ (b) 56

4 Tangent meets the curve again

A curve has equation $y = x^3 - 5x^2 + 2x + 1$.
The tangent to the curve at the point A, where $x = 3$, meets the curve again at the point B.
Find the coordinates of B.

$\frac{dy}{dx} = 3x^2 - 10x + 2$, so the gradient at A is -1, and A = $(3, -11)$
Tangent: $y = -x - 8$
Equate: $x^3 - 5x^2 + 3x + 9 = 0$, which has a repeated root $x = 3$
Sum of the roots = 5, so $3 + 3 + x = 5$ and $x = -1$
$y = -7$
Answer: B $(-1, -7)$

5 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 12 cm, top radius 4 cm and base radius 16 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+12} = \frac{4}{16}$

$16h = 4(h + 12)$, so $12h = 48$ and $h = 4$

Large cone: $\frac{1}{3} \times \pi \times 16^2 \times 16 = \frac{4096}{3}\pi$

Small cone: $\frac{1}{3} \times \pi \times 4^2 \times 4 = \frac{64}{3}\pi$

Frustum = $\frac{4096}{3}\pi - \frac{64}{3}\pi = 1344\pi$ cm^3

Answer: 1344π cm^3

6 Gradient of a chord

A and B are points on the curve $y = x^3 + 5x$.
The x coordinate of A is 2 and the x coordinate of B is 4.
Work out the gradient of the line AB.

At A: $y = 18$. At B: $y = 84$
A is (2, 18) and B is (4, 84)

Gradient = $\frac{84 - 18}{4 - 2} = \frac{66}{2} = 33$

Answer: 33

7 Find unknown coefficients from gradients

The curve $y = ax^2 + bx - 9$ has gradient -9 when $x = -2$ and gradient -11 when $x = -3$.
Find the values of a and b.

$\frac{dy}{dx} = 2ax + b$

$x = -2$: $-4a + b = -9$

$x = -3$: $-6a + b = -11$

Subtract and solve: $a = 1$

$b = -5$

Answer: $a = 1$, $b = -5$

8 Prime factors and indices

Write $8^5 \times 24^2$ as a product of powers of prime numbers.

$8 = 2^3$, $24 = 2^3 \times 3$

$8^5 = 2^{15}$

$24^2 = 2^6 \times 3^2$

Add the powers of each prime: $2^{21} \times 3^2$

Answer: $2^{21} \times 3^2$

9 Range and inverse of a quadratic

$g(x) = x^2 - 10x + 33$ for $x \geq 5$.
(a) Find the range of g.
(b) Find $g^{-1}(x)$.

(a) $g(x) = (x - 5)^2 + 8$

$x \geq 5$ includes the minimum point, so $g(x) \geq 8$

(b) $y = (x - 5)^2 + 8$, so $(x - 5)^2 = y - 8$

$x \geq 5$ means $x - 5$ is positive or zero, so $x = 5 + \sqrt{y - 8}$

$g^{-1}(x) = 5 + \sqrt{x - 8}$

Answer: (a) $g(x) \geq 8$ (b) $g^{-1}(x) = 5 + \sqrt{x - 8}$

10 Tree diagrams and reverse probability

The probability that it rains on a day is 0.5.
If it rains, the probability that Ali is late for school is 0.6.
If it does not rain, the probability that Ali is late is 0.3.
(a) Work out the probability that Ali is late.
(b) Ali is late. Work out the probability that it rained.

(a) P(late) = 0.5 × 0.6 + 0.5 × 0.3 = 0.3 + 0.15 = 0.45
(b) P(rain and late) = 0.3

P(rain | late) = 0.3 ÷ 0.45 = $\frac{2}{3}$

Answer: (a) 0.45 (b) $\frac{2}{3}$