Day 216 **IGCSE 10-a-day: Model Solutions**

1 f′(x) notation

f(x) = $-x^3 + 8x - 4$

(a) Find f′(x).

(b) Work out the value of f′(2).

(a) $-x^3 \to -3x^2$, $8x \to 8$

f′(x) = $-3x^2 + 8$

(b) f′(2) = -4

Answer: (a) f′(x) = $-3x^2 + 8$ (b) -4

2 Column vector arithmetic

$\mathbf{a} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$.

(a) Work out $\mathbf{a} + \mathbf{b}$.

(b) Work out $3\mathbf{a} - 2\mathbf{b}$.

(a) $\begin{pmatrix} 5 \\ -4 \end{pmatrix} + \begin{pmatrix} 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 11 \\ -4 \end{pmatrix}$

(b) $3\mathbf{a} = \begin{pmatrix} 15 \\ -12 \end{pmatrix}$ and $2\mathbf{b} = \begin{pmatrix} 12 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 15 \\ -12 \end{pmatrix} - \begin{pmatrix} 12 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ -12 \end{pmatrix}$

Answer: (a) $\begin{pmatrix} \mathbf{11} \\ \mathbf{-4} \end{pmatrix}$ (b) $\begin{pmatrix} \mathbf{3} \\ \mathbf{-12} \end{pmatrix}$

3 Expanding three brackets

Expand and simplify (x + 4)(x + 5)(x − 3)

Expand the first two brackets: $(x + 4)(x + 5) = x^2 + 9x + 20$

Multiply by (x − 3): $x(x^2 + 9x + 20) - 3(x^2 + 9x + 20)$

$= x^3 + 6x^2 - 7x - 60$

Answer: $x^3 + 6x^2 - 7x - 60$

4 Where is the second derivative zero?

$y = 2x^3 - 18x^2 + 2x - 6$

(a) Find $\frac{d^2y}{dx^2}$.

(b) Find the value of x for which $\frac{d^2y}{dx^2} = 0$, and work out the gradient of the curve at this value of x.

$\frac{dy}{dx} = 6x^2 - 36x + 2$

$\frac{d^2y}{dx^2} = 12x - 36$

$\frac{d^2y}{dx^2} = 0$ gives $12x - 36 = 0$, so $x = 3$

$\frac{dy}{dx} = -52$ when $x = 3$

Answer: (a) $12x - 36$ (b) x = 3, gradient -52

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5 Area of a segment

The circle has centre O and radius 12 cm. A and B are points on the circle and angle AOB = 70°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{70}{360} \times \pi \times 12^2 = 87.965$ cm^2

Area of triangle OAB = $\frac{1}{2} \times 12 \times 12 \times \sin 70° = 67.658$ cm^2

Segment = sector − triangle = 87.965 − 67.658 = 20.307

= 20.3 cm^2 (3 s.f.)

Answer: 20.3 cm^2

6 Using a line of best fit

The scatter graph shows age of car (x) and value (y).
The equation of the line of best fit is $y = -x + 33$.
(a) Estimate y when x = 7.
(b) Explain why an estimate of y when x = 15 may be unreliable.

(a) $y = -1 \times 7 + 33 = 26$

(b) x = 15 is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)

Answer: (a) 26 (b) x = 15 is outside the range of the data

7 Vertical distance between stationary points

The curve $y = x^3 + 6x^2 + 9x + 6$ has two stationary points, A and B.
Work out the vertical distance between A and B.

$\frac{dy}{dx} = 3x^2 + 12x + 9 = 3(x + 3)(x + 1)$

$x = -3$ and $x = -1$

At $x = -3$, $y = 6$; at $x = -1$, $y = 2$

Difference = 6 − 2 = 4

Answer: 4

8 Linear programming: vertices

x and y satisfy $x \geq 0$, $y \geq 0$,
$4x + y \leq 44$ and $2x + 5y \leq 40$.
Find the greatest value of $P = 9x + 7y$ and the values of x and y that give it.

Draw the lines. The feasible region has vertices (0, 0), (0, 8), (10, 4), (11, 0)
Values of P: (0, 0): 0, (0, 8): 56, (10, 4): 118, (11, 0): 99
Greatest value of P is 118 at (10, 4)

Answer: P = 118 when x = 10 and y = 4

9 Speed and a moving train

A train travels at a constant speed of 36 km/h.
It takes 40 seconds to go completely through a tunnel which is 290 m long.
Work out the length of the train, in metres.

Speed = 36 km/h = 36,000 ÷ 3600 = 10 m/s
Distance travelled in 40 s = 10 × 40 = 400 m
This is the length of the tunnel plus the length of the train
Train = 400 − 290 = 110 m

Answer: 110 m

10 Percentage change in area and volume

The side length of a square is decreased by 60%.
Work out the percentage change in the area of the square.

Side length multiplier = 0.4

Area multiplier = $0.4^2 = 0.16$

$0.16 - 1 = -0.84$, so a 84% decrease

Answer: 84% decrease