Day 215 **IGCSE 10-a-day: Model Solutions**

1 Gradient of a chord

A and B are points on the curve $y = 3x^2 + 6x - 2$.
The x coordinate of A is 1 and the x coordinate of B is 4.
Work out the gradient of the line AB.

At A: y = 7. At B: y = 70
A is (1, 7) and B is (4, 70)
Gradient = $\frac{70 - 7}{4 - 1} = \frac{63}{3} = 21$

Answer: 21

2 Area of a triangle using sine

Triangle ABC has AB = 12 cm and BC = 9 cm.
The area of triangle ABC is $27\sqrt{3}$ cm^2.
Angle ABC is acute.
Work out the size of angle ABC.
Do not use a calculator.

Area = $\frac{1}{2} \times AB \times BC \times \sin B$

$27\sqrt{3} = \frac{1}{2} \times 12 \times 9 \times \sin B$

$\sin B = 27\sqrt{3} \div 54 = \frac{\sqrt{3}}{2}$

$\sin 60° = \frac{\sqrt{3}}{2}$, and B is acute, so B = 60°

Answer: 60°

3 Stratified sampling

The table shows the number of students in each of four groups.
(a) A stratified sample of 37 is taken. Work out how many come from Year 7.
(b) A different stratified sample has 33 from Year 9. Work out the size of this sample.

Total = 300 + 150 + 275 + 200 = 925

(a) Year 7: $\frac{300}{925} \times 37 = 12$

(b) Fraction sampled = $33 \div 275 = \frac{3}{25}$, so sample size = $\frac{3}{25} \times 925 = 111$

Answer: (a) 12 (b) 111

4 Rate of change of area or volume

A cylinder has radius r cm and fixed height 3 cm. Its volume is $V = 3\pi r^2$ cm^3.

(a) Find $\frac{dV}{dr}$ when r = 7.

(b) Find the value of r when $\frac{dV}{dr} = 54\pi$.

$\frac{dV}{dr} = 6\pi r$

When r = 7: $\frac{dV}{dr} = 42\pi$

$6\pi r = 54\pi$, so 6r = 54 and r = 9 (lengths are positive)

Answer: (a) 42π (b) r = 9

5 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 6 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 18 ÷ 6 = 3 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 6 \times 18 + 18 \times 10 + \frac{1}{2} \times 18 \times 18 = 54 + 180 + 162$
= 396
Answer: (a) 3 m/s^2 (b) 396 m

6 Venn diagram: shaded region notation

Write down the set notation for the region shaded in the Venn diagram.

Identify which regions are shaded, using the circles P and Q
The shaded region is $Q \cup P'$
Answer: $Q \cup P'$

7 Income tax

In one country, income tax is worked out like this:
The first £12,500 of income is not taxed.
The next £35,000 is taxed at 20%.
All income above that is taxed at 40%.
Ravi pays £13,400 in income tax in a year.
Work out Ravi's income for the year.

Tax on the 20% band = 20% of £35,000 = £7,000
Tax paid at 40% = £13,400 − £7,000 = £6,400
Income taxed at 40% = £6,400 ÷ 0.4 = £16,000
Income = £12,500 + £35,000 + £16,000 = £63,500
Answer: £63,500

8 Second derivative and nature at a given point

$y = -2x^3 - 9x^2 - 12x + 1$
Use the second derivative to show that the curve has a maximum at $x = -1$.

$\frac{dy}{dx} = -6x^2 - 18x - 12$

When $x = -1$: $\frac{dy}{dx} = 0$, so it is a stationary point

$\frac{d^2y}{dx^2} = -12x - 18$

When $x = -1$: $\frac{d^2y}{dx^2} = -6$, which is negative

So there is a maximum at $x = -1$
Answer: maximum at $x = -1$

9 Using a quadratic nth term

The nth term of a sequence is $2n^2 + 2n - 4$.
(a) Work out the 7th term.
(b) One term of the sequence is 416. Work out the value of n for this term.

(a) $2 \times 7^2 + 2 \times 7 - 4 = 108$
(b) $2n^2 + 2n - 4 = 416$ so $2n^2 + 2n - 420 = 0$
Solve (factorise or use the formula): $n = 14$ or $n = -15$. n must be a positive integer, so $n = 14$
Answer: (a) 108 (b) n = 14

10 Open box of fixed surface area

An open tray has a square base of side x cm and height h cm. The total outside surface area is 48 cm^2.

(a) Show that the volume of the box is $V = \frac{1}{4}(48x - x^3)$ cm^3.

(b) Find the maximum volume of the box, proving that it is a maximum.

$x^2 + 4xh = 48$, so $h = \frac{48 - x^2}{4x}$

$V = x^2h = \frac{x(48 - x^2)}{4} = \frac{1}{4}(48x - x^3)$

$\frac{dV}{dx} = -\frac{3}{4}x^2 + 12 = 0$, so $x^2 = 16$ and $x = 4$

$\frac{d^2V}{dx^2} = -\frac{3}{2}x = -6 < 0$, so a maximum

$V = 32$

Answer: 32 cm^3