Day 214 **IGCSE 10-a-day: Model Solutions**

1 Multiplying 2 by 2 matrices

$A = \begin{pmatrix} 1 & 4 \\ 3 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} -3 & -2 \\ -3 & -4 \end{pmatrix}$.

(a) Work out AB.

(b) Work out BA.

(a) AB: row 1 is $1 \times (-3) + 4 \times (-3)$ and $1 \times (-2) + 4 \times (-4)$; row 2 is $3 \times (-3) + (-4) \times (-3)$ and $3 \times (-2) + (-4) \times (-4)$

$AB = \begin{pmatrix} -15 & -18 \\ 3 & 10 \end{pmatrix}$

(b) $BA = \begin{pmatrix} -9 & -4 \\ -15 & 4 \end{pmatrix}$

Answer: (a) AB $= \begin{pmatrix} \mathbf{-15} & \mathbf{-18} \\ \mathbf{3} & \mathbf{10} \end{pmatrix}$ **(b) BA** $= \begin{pmatrix} \mathbf{-9} & \mathbf{-4} \\ \mathbf{-15} & \mathbf{4} \end{pmatrix}$

2 Time zones

A flight leaves London at 16:40 (London time).
It takes 5 hours 15 minutes.
When it is 12:00 in London it is 21:00 in Sydney.
At what time, in Sydney time, does the flight land? Say if it is on a different day.

Arrival in London time = 16:40 + 5 h 15 min = 21:55
Sydney is 9 hours ahead of London, so add 9 hours
06:55 the next day

Answer: 06:55 the next day

3 Tangent at a turning point

The graph of $y = 2x^2 - 16x$ has a turning point at P.
Find the equation of the tangent to the curve at P.

$\frac{dy}{dx} = 4x - 16$

At P, $\frac{dy}{dx} = 0$, so $4x - 16 = 0$ and $x = 4$

$y = -32$ at P

The gradient is 0, so the tangent is horizontal: $y = -32$

Answer: y = −32

4 Inverse proportion in context

10 workers can build a wall in 10 days.
After 6 days, 2 of the workers leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work = 10 × 10 = 100 worker-days
Work done = 10 × 6 = 60, so work left = 40
Workers left = 10 − 2 = 8
Time = 40 ÷ 8 = 5 days

Answer: 5 days

5 Pythagoras with algebra

ABC is a right-angled triangle with sides (x + 1) cm, (2x − 2) cm and (x + 5) cm. The longest side is (x + 5) cm.
(a) Find the value of x.
(b) Work out the area of the triangle.
The diagram is not drawn accurately.

(a) Pythagoras: $(x + 1)^2 + (2x - 2)^2 = (x + 5)^2$
Expand and collect terms: $4x^2 - 16x - 20 = 0$
Factorise: $4(x - 5)(x + 1) = 0$
$x = 5$ or $x = -1$, and x must be positive, so $x = 5$
(b) The two shorter sides are 6 cm and 8 cm
Area $= \frac{1}{2} \times 6 \times 8 = 24$ cm^2

Answer: (a) x = 5 (b) 24 cm^2

6 Points with a given gradient on y = ax + b/x

A curve has equation $y = 2x + \frac{4}{x}$.
Find the coordinates of the points on the curve where the gradient is 1.

$\frac{dy}{dx} = 2 - \frac{4}{x^2}$
Set $\frac{dy}{dx} = 1$: $1 = \frac{4}{x^2}$
$x^2 = 4$, so $x = 2$ or $x = -2$
$y = 6$ when $x = 2$ and $y = -6$ when $x = -2$
Answer: (2, 6) and (−2, −6)

7 Exponential graph from points

The diagram shows part of the graph of $y = ab^x$.
The curve passes through (0, 8) and (4, 648).
(a) Find the values of a and b. (b) Work out the value of y when x = 6.

(a) At $x = 0$: $y = a \times b^0 = a$, so $a = 8$
At $x = 4$: $8 \times b^4 = 648$ so $b^4 = 81$
$b = 3$
(b) $y = 8 \times 3^6 = 5832$
Answer: (a) a = 8, b = 3 (b) 5832

8 Tangent to a circle: proof

Show that the line $y = -x + 14$ is a tangent to the circle $x^2 + y^2 = 98$.
Find the coordinates of the point where the line touches the circle.

Substitute $y = -x + 14$ into $x^2 + y^2 = 98$: $x^2 + (-x + 14)^2 = 98$
Expand: $2x^2 - 28x + 98 = 0$
$b^2 - 4ac = (-28)^2 - 4 \times 2 \times 98 = 784 - 784 = 0$
$b^2 - 4ac = 0$, so there is exactly one point of intersection: the line is a tangent
$x = \frac{-(-28)}{2 \times 2} = 7$, and $y = -7 + 14 = 7$. The point is (7, 7)
Answer: (7, 7)

9 Counting factors

$N = 2^4 \times 3^3 \times 7$
How many of the factors of N are odd numbers?

An odd factor contains no 2s
Ignore the 2 and count the factors of the rest: $4 \times 2 = 8$
Answer: 8

10 Where two tangents meet

The curve C has equation $y = 2x^2 + 6x + 6$.
The points A and B on C have x coordinates 0 and 2.
The tangents to C at A and at B meet at the point T. Find the coordinates of T.

$\frac{dy}{dx} = 4x + 6$. At A: gradient 6, A = (0, 6)
Tangent at A: $y = 6x + 6$. At B: gradient 14, B = (2, 26)
Tangent at B: $y = 14x - 2$
Equate: $6x + 6 = 14x - 2$, so $x = 1$ and $y = 12$
Answer: T (1, 12)