Day 213 **IGCSE 10-a-day: Model Solutions**

1 Vector between two points

A is the point (7, −2) and B is the point (7, 2).
(a) Write down the vector **AB**.
(b) Find |**AB**|, giving the answer as a surd in simplest form if it is not exact.

(a) $\mathbf{AB} = \begin{pmatrix}7\\2\end{pmatrix} - \begin{pmatrix}7\\-2\end{pmatrix} = \begin{pmatrix}0\\4\end{pmatrix}$

(b) $|\mathbf{AB}| = \sqrt{(0)^2 + (4)^2} = \sqrt{16} = 4$

Answer: (a) $\begin{pmatrix}\mathbf{0}\\\mathbf{4}\end{pmatrix}$ **(b) 4**

2 Arithmetic sequences

The 5th term of an arithmetic sequence is 45.
The 11th term of the sequence is 81.
(a) Find an expression for the nth term of the sequence.
(b) Is 158 a term of the sequence? You must show your working.

(a) Common difference = (81 − 45) ÷ (11 − 5) = 6
nth term = 6n + c. For n = 5: 6 × 5 + c = 45, so c = 15
nth term = 6n + 15

(b) Solve 6n + 15 = 158: 6n = 143, so n = $\frac{143}{6}$

n is not a whole number, so 158 is not a term
Answer: (a) 6n + 15 (b) No

3 Percentages

Q is 10% less than R.
R is 60% more than B.
Work out Q as a percentage of B.

Q = 0.9 × R and R = 1.6 × B
Q = 0.9 × 1.6 × B = 1.44 × B
1.44 × 100 = 144%
Answer: 144%

4 Perpendicular bisector

A is the point (0, 0) and B is the point (4, −2).
Find an equation of the perpendicular bisector of AB. Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = (2, −1)

Gradient of AB = $-\frac{1}{2}$

Perpendicular gradient = 2
y − (−1) = 2(x − 2)
Rearrange to integer coefficients: 2x − y = 5
Answer: 2x − y = 5

5 Cosine rule: finding a side

Work out the length of BC.
Give your answer correct to 3 significant figures. (Calculator)

$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos A$
$BC^2 = 6^2 + 6.9^2 - 2 \times 6 \times 6.9 \times \cos 98°$
$BC^2 = 95.134$
BC = 9.75
Answer: 9.75 cm

6 Increasing and decreasing with powers

$f(x) = 2x + \frac{8}{x}$ for $x > 0$

(a) Find the range of values of x for which f is a decreasing function.
(b) Write down the least value of f(x).

$f'(x) = 2 - 8x^{-2} = 2 - \frac{8}{x^2}$

f is decreasing when $f'(x) < 0$: $2 < \frac{8}{x^2}$, so $x^2 < 4$

With $x > 0$ this gives $0 < x < 2$
f decreases then increases, so the least value is at $x = 2$: $f(2) = 8$
Answer: (a) $0 < x < 2$ (b) 8

7 Cumulative frequency graphs

The cumulative frequency graph gives information about the masses of 80 apples picked from a tree.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 80 had a mass greater than 65.

(a) Median at cumulative frequency 40, so median = 50
Lower quartile at 20, so LQ = 40. Upper quartile at 60, so UQ = 60
Interquartile range = 60 − 40 = 20
(b) At 65, cumulative frequency = 67
Number greater = 80 − 67 = 13
Answer: (a) median 50, IQR 20 (b) 13

8 Points with a given gradient on y = ax + b/x

A curve has equation $y = -4x - \frac{3}{x}$.

Find the coordinates of the points on the curve where the gradient is −1.

$\frac{dy}{dx} = -4 + \frac{3}{x^2}$

Set $\frac{dy}{dx} = -1$: $-3 = \frac{-3}{x^2}$

$x^2 = 1$, so $x = 1$ or $x = -1$
$y = -7$ when $x = 1$ and $y = 7$ when $x = -1$
Answer: (1, −7) and (−1, 7)

9 Percentages with an unknown rate

A population is decreased by x% in one year and then decreased by a further x% the next year.
Over the two years the population decreases by 75%.
Work out the value of x.

Yearly multiplier = $1 - \frac{x}{100}$

Two years: $(1 - \frac{x}{100})^2 = 0.25$

Take the square root: $1 - \frac{x}{100} = 0.5$

$\frac{x}{100} = 0.5$, so $x = 50$
Answer: x = 50

10 Where two tangents meet

The curve C has equation $y = x^2 + 6x + 3$.
The points A and B on C have x coordinates 0 and 4.
The tangents to C at A and at B meet at the point T. Find the coordinates of T.

$\frac{dy}{dx} = 2x + 6$. At A: gradient 6, A = (0, 3)
Tangent at A: $y = 6x + 3$. At B: gradient 14, B = (4, 43)
Tangent at B: $y = 14x - 13$
Equate: $6x + 3 = 14x - 13$, so $x = 2$ and $y = 15$
Answer: T (2, 15)