Day 212 **IGCSE 10-a-day: Model Solutions**

1 Tangent at a turning point

The graph of $y = 3x^2 + 24x - 4$ has a turning point at P.
Find the equation of the tangent to the curve at P.

$\frac{dy}{dx} = 6x + 24$

At P, $\frac{dy}{dx} = 0$, so $6x + 24 = 0$ and $x = -4$

$y = -52$ at P
The gradient is 0, so the tangent is horizontal: $y = -52$
Answer: $y = -52$

2 Magnitude of a vector

$\mathbf{v} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$.
Find $|\mathbf{v}|$. Give your answer as a surd in its simplest form.

$|\mathbf{v}| = \sqrt{(5)^2 + (8)^2} = \sqrt{25 + 64}$
$= \sqrt{89}$
Answer: $\sqrt{89}$

3 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the adjacent side AB and the known side is the hypotenuse AC, so use cos 60°

$\cos 60° = \frac{1}{2}$

$x = 5 \times \cos 60° = 5 \times \frac{1}{2}$

$x = \frac{5}{2}$

Answer: $x = \frac{5}{2}$

4 Increasing or decreasing at a point

$y = 2x^3 - 24x - 8$
At the point where x = 1, is the curve increasing or decreasing? You must show your working.

$\frac{dy}{dx} = 6x^2 - 24$

When x = 1: $\frac{dy}{dx} = -18$

-18 is negative, so the curve is decreasing
Answer: decreasing (gradient -18)

5 Pressure

A metal crate in the shape of a cuboid weighs 540 N.
Its smallest face has area 300 cm^2. Its largest face has area 600 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the metal crate can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 300 cm^2 = 0.03 m^2 and 600 cm^2 = 0.06 m^2
(a) Greatest pressure is on the smallest face: 540 ÷ 0.03 = 18,000 N/m^2
(b) Least pressure is on the largest face: 540 ÷ 0.06 = 9,000 N/m^2
Answer: (a) 18,000 N/m^2 (b) 9,000 N/m^2

6 Perpendicular lines

The diagram shows the straight line L, which passes through the points A and B.
The line M is perpendicular to L and passes through B.
Find the equation of M in the form y = mx + c.

Gradient of L = $\frac{-4-0}{0-6} = \frac{2}{3}$

Perpendicular gradient = $-1 \div \frac{2}{3} = -\frac{3}{2}$

M passes through B(6, 0): $0 = (-\frac{3}{2}) \times 6 + c$, so c = 9

$y = -\frac{3}{2}x + 9$

Answer: $y = -\frac{3}{2}x + 9$

7 Find constants from a turning point

The diagram shows a sketch of the curve $y = ax^2 + bx - 3$.
The curve has a turning point at M(2, 1).
Find the values of a and b.

$\frac{dy}{dx} = 2ax + b = 0$ when x = 2, so b = −4a

The point (2, 1) is on the curve: 4a + 2b − 3 = 1
Substitute b = −4a: −4a = 4, so a = −1 and b = 4
Answer: a = −1, b = 4

8 Tree diagrams and reverse conditional probability

Machine A makes 25% of the items and machine B makes the rest.
2% of items from A are faulty and 9% of items from B are faulty.
An item is chosen at random.
(a) Work out the probability that the item is faulty.
(b) The item is faulty. Work out the probability that it came from machine A.

(a) 0.25 × 0.02 = 0.005 and 0.75 × 0.09 = 0.0675
P(faulty) = 0.005 + 0.0675 = 0.0725
(b) $\frac{0.005}{0.0725} = \frac{2}{29}$

Answer: (a) 0.0725 (b) $\frac{2}{29}$

9 Prime factors and indices

Write $6^2 \times 8^4$ as a product of powers of prime numbers.

$6 = 2 \times 3$, $8 = 2^3$
$6^2 = 2^2 \times 3^2$
$8^4 = 2^{12}$
Add the powers of each prime: $2^{14} \times 3^2$
Answer: $2^{14} \times 3^2$

10 Perpendicular bisector

A is the point (4, 0) and B is the point (6, −6).
Find an equation of the perpendicular bisector of AB.
Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = $(\frac{4+6}{2}, \frac{0+(-6)}{2}) = (5, -3)$

Gradient of AB = $\frac{-6-0}{6-4} = -3$

The perpendicular gradient is $-1 \div (-3) = \frac{1}{3}$

Using the midpoint: $y + 3 = \frac{1}{3}(x - 5)$

Multiply by 3 and rearrange: $-x + 3y = -14$
Multiply by −1: $x - 3y = 14$
Answer: $x - 3y = 14$