Day 211 **IGCSE 10-a-day: Model Solutions**

1 Differentiating polynomials

Differentiate $6x^4 - 8x^3 - 8x^2 - 8x$ with respect to x.

Differentiate each term: $x^n \to nx^{n-1}$ and a constant $\to 0$

$6x^4 \to 24x^3$, $-8x^3 \to -24x^2$, $-8x^2 \to -16x$, $-8x \to -8$

$\frac{dy}{dx} = 24x^3 - 24x^2 - 16x - 8$

Answer: $\frac{\mathbf{dy}}{\mathbf{dx}} = \mathbf{24x^3 - 24x^2 - 16x - 8}$

2 Collinear points with vectors

$\mathbf{OA} = \mathbf{b}$, $\mathbf{OB} = 3\mathbf{a} + 5\mathbf{b}$ and $\mathbf{OC} = 9\mathbf{a} + 13\mathbf{b}$.

Show that A, B and C lie on a straight line.

$\mathbf{AB} = \mathbf{OB} - \mathbf{OA} = 3\mathbf{a} + 4\mathbf{b}$

$\mathbf{AC} = \mathbf{OC} - \mathbf{OA} = 9\mathbf{a} + 12\mathbf{b}$

$\mathbf{AC} = 3\mathbf{AB}$

AC is a multiple of AB and they share the point A, so A, B and C are collinear

Answer: AC = 3AB, so A, B, C are collinear

3 Projectile height

A ball is thrown upwards. Its height above the ground, h metres, after t seconds is given by

$h = -4t^2 + 16t + 2$

(a) Work out the greatest height reached by the ball.

(b) Work out the velocity of the ball when t = 3. State whether it is moving up or down.

$\frac{dh}{dt} = -8t + 16 = 0$ gives $t = 2$

$\frac{d^2h}{dt^2} = -8 < 0$ so maximum. $h = 18$

When t = 3: $\frac{dh}{dt} = 16 - 8 \times 3 = -8$

Answer: (a) 18 m (b) −8 m/s, moving down

4 Cyclic quadrilaterals

A, B, C and D are points on a circle. Angle DAB = (3x + 10)°, angle BCD = (5x + 42)° and angle ABC = (3x + 25)°.

(a) Work out the value of x.

(b) Work out the size of angle ADC.

The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°

(3x + 10) + (5x + 42) = 180

8x + 52 = 180, so 8x = 128

x = 16

(b) Angle ABC = 3 × 16 + 25 = 73°

Angle ADC = 180 − 73 = 107°

Answer: (a) x = 16 (b) 107°

5 Stationary point of a quartic

$y = -x^4 - 4x$

Find the coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = -4x^3 - 4$

$\frac{dy}{dx} = 0$: $-4x^3 - 4 = 0$, so $x^3 = -1$

$x = -1$ and $y = 3$

$\frac{d^2y}{dx^2} = -12x^2$

At $x = -1$: $\frac{d^2y}{dx^2} = -12$, so a maximum

Answer: $(-1, 3)$, maximum

6 Rates of work

Pipe A can fill an empty pool in 6 minutes.
Pipe B can fill an empty pool in 30 minutes.
Both are turned on at the same time.
How long do they take to fill an empty pool?

In 1 minute: Pipe A does $\frac{1}{6}$ and Pipe B does $\frac{1}{30}$

Together: $\frac{1}{6} + \frac{1}{30} = \frac{36}{180}$

Time = $\frac{180}{36}$ = 5 minutes

Answer: 5 minutes

7 Where a line meets the axes

The line L has equation $12x + 5y = 120$.
L crosses the x-axis at the point A and the y-axis at the point B.
(a) Find the coordinates of A and B.
(b) Work out the area of triangle OAB, where O is the origin.

(a) At A, $y = 0$: $12x = 120$, so $x = 10$. A is (10, 0)
At B, $x = 0$: $5y = 120$, so $y = 24$. B is (0, 24)
(b) OA = 10 and OB = 24, and angle AOB = 90°

Area = $\frac{1}{2} \times 10 \times 24 = 120$

Answer: (a) A(10, 0), B(0, 24) (b) 120

8 Histogram with unequal class widths

The histogram shows the ages, in years, of some visitors.
The bar for 25 to 30 represents 10 visitors.
(a) Work out the frequency of the class 10 to 20.
(b) Work out the total frequency.

Frequency = k × height × width. For 25 to 30: $10 = k \times 2 \times 5$ so $k = 1$
(a) 10 to 20: $1 \times 6 \times 10 = 60$
(b) Frequencies: 60, 60, 20, 10
Total = 150
Answer: (a) 60 (b) 150

9 Surds with unknowns

Given that $(a + \sqrt{6})^2 = 15 + 6\sqrt{6}$,
where a is a positive integer, find the value of a.

$(a + \sqrt{6})^2 = a^2 + 2a\sqrt{6} + 6$

Compare the $\sqrt{6}$ terms: $2a = 6$, so $a = 3$

Check the integer terms: $a^2 + 6 = 9 + 6 = 15$

Answer: a = 3

10 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 2 : 1. N is on OB with ON : NB = 3 : 1. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{2}{3}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{2}{3}\mathbf{a} = \frac{2}{3}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{2}{3}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{3}{4}t\mathbf{b}$

Equate: $\frac{2}{3}s = 1 - t$ and $1 - s = \frac{3}{4}t$

Solving: $s = \frac{1}{2}$ and $t = \frac{2}{3}$

OX = $\frac{1}{3}\mathbf{a} + \frac{1}{2}\mathbf{b}$

Answer: (a) $\frac{2}{3}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{1}{3}\mathbf{a} + \frac{1}{2}\mathbf{b}$