Day 210 **IGCSE 10-a-day: Model Solutions**

1 Stratified sampling

The table shows the number of students in each of four groups.
(a) A stratified sample of 28 is taken. Work out how many come from Year 8.
(b) A different stratified sample has 24 from Year 7. Work out the size of this sample.

Total = 160 + 100 + 220 + 80 = 560

(a) Year 8: $\frac{100}{560} \times 28 = 5$

(b) Fraction sampled = $24 \div 160 = \frac{3}{20}$, so sample size = $\frac{3}{20} \times 560 = 84$

Answer: (a) 5 (b) 84

2 Equations with fractions

Solve $\frac{x-4}{2} + \frac{x-6}{6} = -7$

Multiply every term by 6: $3(x-4) + (x-6) = -42$
Expand: $4x - 18 = -42$
$4x = -24$
$x = -6$
Answer: $x = -6$

3 Maximum speed from velocity

A particle moves in a straight line. Its velocity, v m/s, at time t seconds is given by
$v = 4t - t^2$ for $0 \leq t \leq 4$
Find the time at which the speed of the particle is greatest, and this greatest speed.

$a = \frac{dv}{dt} = -2t + 4$
Greatest speed when $a = 0$: $-2t + 4 = 0$, so $t = 2$
$\frac{d^2v}{dt^2} = -2 < 0$, so this is a maximum
$v = 4 \times 2 - 1 \times 2^2 = 4$
Answer: t = 2 s, greatest speed 4 m/s

4 Cosine rule: finding a side

Work out the length of BC.
Give your answer correct to 3 significant figures. (Calculator)

$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos A$
$BC^2 = 7.8^2 + 12.9^2 - 2 \times 7.8 \times 12.9 \times \cos 50°$
$BC^2 = 97.895$
$BC = 9.89$
Answer: 9.89 cm

5 Ratio and algebra

In a bag, the ratio of the number of red counters to the number of blue counters is 2 : 3.
Then 6 red counters are removed and 10 blue counters are removed.
The ratio is now 8 : 11.
How many red counters were there at the start?

Let the numbers at the start be 2k and 3k
$(2k - 6) : (3k - 10) = 8 : 11$, so $11(2k - 6) = 8(3k - 10)$
$22k - 66 = 24k - 80$
$-2k = -14$, so $k = 7$
Number of red counters at the start: $2 \times 7 = 14$
Answer: 14

6 Simple and compound interest

Tom invests £15,000 for 5 years at 1.5% per year simple interest.
Lena invests £15,000 for 5 years at 1.5% per year compound interest.
Work out how much more interest Lena earns than Tom.
Give your answer to the nearest penny. (Calculator)

Simple interest: £15,000 × 1.5% × 5 = £1,125
Compound: £15,000 × 1.015^5 = £16,159.26
Compound interest = £16,159.26 − £15,000 = £1,159.26
Difference = £1,159.26 − £1,125 = £34.26
Answer: £34.26

7 3D problems: pyramids

VABCD is a pyramid on a square base. V is directly above the centre M.
(a) Work out the height VM of the pyramid.
(b) Work out the volume of the pyramid.
The diagram is not drawn accurately.

(a) $AC^2 = 12^2 + 12^2 = 288$, so $AM^2 = (AC \div 2)^2 = 72$
$VM^2 = VA^2 - AM^2 = 9^2 - 72 = 9$
VM = 3 cm
(b) Volume = $\frac{1}{3} \times 12^2 \times 3 = 144$ cm^3
Answer: (a) 3 cm (b) 144 cm^3

8 Tangent parallel to a chord

The diagram shows part of the curve $y = -2x^2 - 4x + 1$. A and B are the points on the curve with x coordinates −2 and 2. C is the point on the curve between A and B at which the tangent is parallel to AB. Find the coordinates of C.

A = (−2, 1) and B = (2, −15)
Gradient of AB = $\frac{-15 - 1}{2 - (-2)} = -4$
$\frac{dy}{dx} = -4x - 4 = -4$, so x = 0
y = 1
Answer: C (0, 1)

9 Show that: consecutive terms of a quadratic sequence

The nth term of a sequence is $3n^2 + 6n + 5$.
(a) Show that the difference between the (n + 1)th term and the nth term is 6n + 9.
(b) Two consecutive terms of the sequence differ by 129. Find the value of n for the first of these two terms.

(a) (n + 1)th term = $3(n + 1)^2 + 6(n + 1) + 5 = 3n^2 + 12n + 14$
Subtract the nth term: $3n^2 + 12n + 14 - (3n^2 + 6n + 5) = 6n + 9$
(b) 6n + 9 = 129 so 6n = 120
n = 20
Answer: (b) n = 20

10 Curve touching the x-axis

The curve $y = -x^3 + 12x + k$, where k is a constant, has the x-axis as a tangent.
Find the two possible values of k.

$\frac{dy}{dx} = -3x^2 + 12 = 0$ gives x = −2 or x = 2
The x-axis is a tangent when a turning point has y = 0
x = −2: −16 + k = 0, so k = 16
x = 2: 16 + k = 0, so k = −16
Answer: k = 16 or k = −16