Day 21 **IGCSE 10-a-day: Model Solutions**

1 Arithmetic sequences

The 6th term of an arithmetic sequence is 44.
The 11th term of the sequence is 59.
(a) Find an expression for the nth term of the sequence.
(b) Is 102 a term of the sequence? You must show your working.

(a) Common difference = $(59 - 44) \div (11 - 6) = 3$
nth term = $3n + c$. For $n = 6$: $3 \times 6 + c = 44$, so $c = 26$
nth term = $3n + 26$

(b) Solve $3n + 26 = 102$: $3n = 76$, so $n = \frac{76}{3}$

n is not a whole number, so 102 is not a term
Answer: (a) $3n + 26$ (b) No

2 Increasing and decreasing intervals

$y = x^3 + 3x^2 - 8$
Find the range of values of x for which y is increasing.

$\frac{dy}{dx} = 3x^2 + 6x = 3x(x + 2)$

$\frac{dy}{dx} = 0$ when $x = -2$ and $x = 0$

The graph of $\frac{dy}{dx}$ against x is a U-shaped quadratic

y is increasing when $\frac{dy}{dx} > 0$ and decreasing when $\frac{dy}{dx} < 0$

So y is increasing when $x < -2$ or $x > 0$
Answer: $x < -2$ or $x > 0$

3 Solve a composite function equation

$f(x) = 2x - 1$ and $g(x) = x^2 + 3$.
Solve $gf(x) = 39$.

$gf(x) = (2x - 1)^2 + 3$

$(2x - 1)^2 = 36$, so $2x - 1 = \pm 6$

$2x - 1 = 6$ gives $x = \frac{7}{2}$; $2x - 1 = -6$ gives $x = -\frac{5}{2}$

Answer: $x = -\frac{5}{2}$ or $x = \frac{7}{2}$

4 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the length of the chord AB.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AM^2 = OA^2 - OM^2 = 10^2 - 6^2 = 64$
AM = 8 cm
AB = $2 \times 8 = 16$ cm

(b) $\sin(\text{angle AOM}) = \frac{8}{10}$

Angle AOM = 53.13°, so angle AOB = $2 \times 53.13 = 106.3°$
Answer: (a) 16 cm (b) 106.3°

5 Compound interest

Nadia invests £9,500. The rates of interest for the next 3 years are 4%, 2% and 4%.
Compound interest is paid. Work out the value of the investment after 3 years.
Give your answer to the nearest penny. (Calculator)

Multipliers: 1.04, 1.02 and 1.04
9500 × 1.04 × 1.02 × 1.04 = 10480.70
To the nearest penny: £10,480.70
Answer: £10,480.70

6 Percentages

B is 30% more than S.
S is 25% more than A.
Work out B as a percentage of A.

B = 1.3 × S and S = 1.25 × A
B = 1.3 × 1.25 × A = 1.625 × A
1.625 × 100 = 162.5%
Answer: 162.5%

7 Fastest rate of growth

The number of bacteria in a culture, N, after t hours is modelled by
$N = -t^3 + 9t^2 + 8t + 200$ for $0 \leq t \leq 12$
Find the time at which N is increasing most rapidly, and the rate of increase at this time.

$\frac{dN}{dt} = -3t^2 + 18t + 8$

Rate is greatest when $\frac{d^2N}{dt^2} = -6t + 18 = 0$, so t = 3

$\frac{d^2N}{dt^2}$ is decreasing (gradient −6), so the rate is a maximum

$\frac{dN}{dt} = 35$ when t = 3

Answer: t = 3, rate 35 per hour

8 Magnitude of a vector

$\mathbf{v} = \begin{pmatrix} k \\ 15 \end{pmatrix}$ and $|\mathbf{v}| = 17$.

Find the two possible values of k.

$k^2 + 225 = 289$, so $k^2 = 64$
k = 8 or k = −8
Answer: k = 8 or k = −8

9 Estimated mean with an unknown frequency

The table gives information about the masses of some eggs.
An estimate for the mean is 43.5.
Work out the value of x.

Mid-points: 10, 30, 50, 70
Estimated mean = Σfm ÷ Σf, so (1290 + 50x) ÷ (31 + x) = 43.5
1290 + 50x = 43.5(31 + x) = 1348.5 + 43.5x
(50 − 43.5)x = 1348.5 − 1290
6.5x = 58.5, so x = 9
Answer: x = 9

10 Show particle never at rest

A particle moves in a straight line. After t seconds its displacement from a fixed point is s metres, where $s = 2t^3 - 12t^2 + 32t - 2$

(a) Show that the particle is never at rest.

(b) Find the least speed of the particle and the time it occurs.

$v = \frac{ds}{dt} = 6t^2 - 24t + 32 = 6(t - 2)^2 + 8$

A square is never negative, so $v \geq 8 > 0$ and P is never at rest

Least value of v occurs when t = 2, giving 8

Answer: (b) 8 m/s when t = 2 s