Day 209 **IGCSE 10-a-day: Model Solutions**

1 Equations with fractions

Solve $\frac{x+1}{2} - \frac{x-7}{5} = 4$

Multiply every term by 10: $5(x + 1) - 2(x - 7) = 40$
Expand: $3x + 19 = 40$
$3x = 21$
$x = 7$
Answer: x = 7

2 Time zones

A flight leaves London at 10:45 (London time).
It takes 11 hours 35 minutes.
When it is 12:00 in London it is 14:00 in Cairo.
At what time, in Cairo time, does the flight land? Say if it is on a different day.

Arrival in London time = 10:45 + 11 h 35 min = 22:20
Cairo is 2 hours ahead of London, so add 2 hours
00:20 the next day
Answer: 00:20 the next day

3 Where is the second derivative zero?

$y = 2x^3 - 18x^2 + 2x - 7$

(a) Find $\frac{d^2y}{dx^2}$.

(b) Find the value of x for which $\frac{d^2y}{dx^2} = 0$, and work out the gradient of the curve at this value of x.

$\frac{dy}{dx} = 6x^2 - 36x + 2$

$\frac{d^2y}{dx^2} = 12x - 36$

$\frac{d^2y}{dx^2} = 0$ gives $12x - 36 = 0$, so $x = 3$

$\frac{dy}{dx} = -52$ when $x = 3$

Answer: (a) 12x − 36 (b) x = 3, gradient −52

4 Estimating the mean from grouped data

The table shows the heights of 52 plants.
(a) Write down the modal class.
(b) Work out an estimate for the mean. Give your answer to 1 decimal place. (Calculator)

(a) Highest frequency is 16: class $10 < x \leq 15$
(b) Midpoints: 2.5, 7.5, 12.5, 17.5
$\Sigma fx = 10 \times 2.5 + 13 \times 7.5 + 16 \times 12.5 + 13 \times 17.5 = 550$
Mean $\approx 550 \div 52 = 10.6$
Answer: (a) $10 < x \leq 15$ (b) 10.6

5 Tangent to a curve with negative or fractional powers

A curve has equation $y = 3x^2 - \frac{3}{x}$.

Find the equation of the tangent to the curve at the point where x = 1.

$\frac{dy}{dx} = 6x + 3x^{-2}$

At x = 1: y = 0 and gradient = 9
$y = 9(x - 1)$
$y = 9x - 9$
Answer: y = 9x − 9

6 Geometric sequences

A geometric sequence has term 4 equal to 54 and term 7 equal to 1458.
Find the common ratio and the first term.

Term 4 = $a r^3$, term 7 = $a r^6$

Divide: $r^3 = 1458 \div 54 = 27$

$r = 3$

$a = 54 \div 3^3 = 2$

Answer: r = 3, first term = 2

7 Fastest rate of growth

The number of bacteria in a culture, N, after t hours is modelled by
$N = -t^3 + 6t^2 + 8t + 1000$ for $0 \leq t \leq 8$
Find the time at which N is increasing most rapidly, and the rate of increase at this time.

$\frac{dN}{dt} = -3t^2 + 12t + 8$

Rate is greatest when $\frac{d^2N}{dt^2} = -6t + 12 = 0$, so $t = 2$

$\frac{d^2N}{dt^2}$ is decreasing (gradient -6), so the rate is a maximum

$\frac{dN}{dt} = 20$ when $t = 2$

Answer: t = 2, rate 20 per hour

8 Shaded areas: exact answers

The diagram shows a circle inside a square of side 12 cm.
The circle touches all four sides of the square.
Work out the area of the shaded region. Give your answer in terms of π.

Area of square = $12 \times 12 = 144$

Radius of circle = $12 \div 2 = 6$ cm

Area of circle = $\pi \times 6^2 = 36\pi$

Shaded area = $144 - 36\pi$ cm^2

Answer: $(144 - 36\pi)$ cm^2

9 Functions that are their own inverse

$f(x) = \frac{-x + 6}{2x + 1}$ where $x \neq -\frac{1}{2}$

(a) Show that ff(x) = x.

(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by (2x + 1)

$ff(x) = \frac{(-1)(-x + 6) + 6(2x + 1)}{2(-x + 6) - (-1)(2x + 1)}$

Top = $x - 6 + 12x + 6 = 13x$

Bottom = $-2x + 12 + 2x + 1 = 13$

$ff(x) = \frac{13x}{13} = x$

(b) ff(x) = x, so f is its own inverse: $f^{-1}(x) = \frac{-x + 6}{2x + 1}$

Answer: (b) $f^{-1}(x) = \frac{-x + 6}{2x + 1}$

10 Speed and a moving train

A train travels at a constant speed of 72 km/h.
It takes 25 seconds to cross completely over a bridge which is 320 m long.
Work out the length of the train, in metres.

Speed = 72 km/h = 72,000 ÷ 3600 = 20 m/s
Distance travelled in 25 s = 20 × 25 = 500 m
This is the length of the bridge plus the length of the train
Train = 500 − 320 = 180 m
Answer: 180 m