Day 208 **IGCSE 10-a-day: Model Solutions**

1 Direct variation

M is directly proportional to the square root of n.
When n = 9, M = 45.
(a) Find a formula for M in terms of n.
(b) Find M when n = 36.

$M = k\sqrt{n}$
$45 = k \times \sqrt{9}$ so $k = 15$
$M = 15\sqrt{n}$
$M = 15 \times 6 = 90$

Answer: (a) $M = 15\sqrt{n}$ (b) $M = 90$

2 Normal meets the curve again (quadratic)

The normal to the curve $y = x^2 + 2x$ at the point P where $x = -2$ meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = 2x + 2$: at P, gradient $= -2$ and $y = 0$

Normal: $y = \frac{1}{2}x + 1$

Intersection: $x^2 + \frac{3}{2}x - 1 = 0$

One root is $x = -2$, and the sum of the roots is $-\frac{3}{2}$

So $x_Q = -\frac{3}{2} - (-2) = \frac{1}{2}$

$y = \frac{5}{4}$

Answer: $(\frac{1}{2}, \frac{5}{4})$

3 Solving quadratics by factorising

Solve $6x^2 - 23x = -20$

Rearrange so that one side is 0: $6x^2 - 23x + 20 = 0$
Factorise: $(3x - 4)(2x - 5) = 0$
$3x - 4 = 0$ or $2x - 5 = 0$
$x = \frac{4}{3}$ or $x = \frac{5}{2}$

Answer: $x = \frac{4}{3}$ or $x = \frac{5}{2}$

4 Speed-time graphs

The graph shows the speed of a car during 15 seconds.
(a) Work out the acceleration during the first 5 seconds.
(b) Work out the average speed over the whole 15 seconds.

(a) Acceleration = change in speed ÷ time = $(10 - 0) \div 5 = 2$ m/s^2
(b) Distance = area under the graph
Trapezium: $\frac{0 + 10}{2} \times 5 = 25$; rectangle: $10 \times 6 = 60$; triangle: $\frac{1}{2} \times 4 \times 10 = 20$
Total = 105 m, so average speed = $105 \div 15 = 7$ m/s

Answer: (a) 2 m/s^2 (b) 7 m/s

5 Percentage change

The mass of a sample is increased by 150%.
By what percentage must the new value be decreased to get back to the original value?

Let the original be 100. After a 150% increase it is 250.
To return to 100, decrease by 150

Percentage decrease = $\frac{150}{250} \times 100 = 60\%$

Answer: 60%

6 Maximum profit

A company makes and sells x thousand items. All amounts are in thousands of pounds.
The revenue is R = x(36 − 2x) and the cost is C = 4 + 4x.
(a) Show that the profit P = R − C is P = $-2x^2 + 32x - 4$.
(b) Find the number of items that must be sold for the profit to be a maximum, and this maximum profit.

$P = 36x - 2x^2 - (4 + 4x) = -2x^2 + 32x - 4$

$\frac{dP}{dx} = -4x + 32 = 0$ gives $x = 8$

$\frac{d^2P}{dx^2} = -4 < 0$, so a maximum

$P = 124$

Answer: 8000 items, maximum profit £124000

7 Transformations of graphs

The graph of y = f(x) has a minimum point at (−2, −5).
Write down the coordinates of the turning point of the graph of
(a) y = f(x) + 5
(b) y = f(2x) − 3

(a) Start with the turning point (−2, −5)
Apply the transformation to the point: (−2, 0)
(b) f(2x) changes the x-coordinate to −1, and f(x) − 3 changes the y-coordinate to −8
Turning point: (−1, −8)
Answer: (a) (−2, 0) (b) (−1, −8)

8 Greatest and least values on an interval

$f(x) = -x^3 - 3x^2 + 9x - 5$ for $-4 \le x \le 2$.
Find the greatest value and the least value of f(x) on this interval.

$f'(x) = -3x^2 - 6x + 9 = 0$ gives $x = -3$ or $x = 1$ (both in the interval)
$f(-4) = -25$, $f(-3) = -32$, $f(1) = 0$, $f(2) = -7$
Compare the turning points and both end values
Answer: Greatest 0, least −32

9 Quadrilateral: cosine rule, then area

In quadrilateral ABCD, AB = 6 cm, BC = 8 cm, CD = 11 cm,
angle ABC = 104° and angle ACD = 64°.
Work out the area of ABCD. Give your answer correct to 3 significant figures. (Calculator)

$AC^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \times \cos 104°$ so AC = 11.101 cm
Area ABC = ½ × 6 × 8 × sin 104° = 23.29
Area ACD = ½ × 11.101 × 11 × sin 64° = 54.87
Total area = 23.29 + 54.87 = 78.2
Answer: 78.2 cm^2

10 Equations with indices

Solve $9^{x-1} = 27^{2x}$

Write both sides as powers of 3: $(3^2)^{x-1} = (3^3)^{2x}$

$3^{2x-2} = 3^{6x}$

Equate the powers: $2x - 2 = 6x$

$-4x = 2$

$x = -\frac{1}{2}$

Answer: $x = -\frac{1}{2}$