Day 207 **IGCSE 10-a-day: Model Solutions**

1 Rearrange then differentiate

$y = \frac{2x^3 - 3x}{x}$

Find $\frac{dy}{dx}$.

Divide each term by x: $y = 2x^2 - 3$

$\frac{dy}{dx} = 4x$

Answer: $\frac{dy}{dx} = 4x$

2 Simultaneous equations

Solve the simultaneous equations

$-5x + 6y = -10$

$6x - 5y = 1$

To match the y terms: (1) × 5 gives $-25x + 30y = -50$, and (2) × 6 gives $36x - 30y = 6$

Add the equations to eliminate y: $11x = -44$

$x = -4$

Substitute $x = -4$ into (1): $6y + 20 = -10$

$6y = -30$, so $y = -5$

Answer: $x = -4$, $y = -5$

3 Particle at rest

A particle P moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where

$s = 2t^3 - 12t^2 + 18t + 2$ $(t \geq 0)$

Find the distance between the two positions where P is at rest.

$v = \frac{ds}{dt} = 6t^2 - 24t + 18$

P is at rest when $v = 0$: $6t^2 - 24t + 18 = 0$

$6(t - 1)(t - 3) = 0$, so $t = 1$ or $t = 3$

$s(1) = 10$ and $s(3) = 2$

Distance $= |2 - 10| = 8$

Answer: 8 m

4 Points satisfying inequalities

R is the region that satisfies all of these inequalities:

$y < 2x + 3$ $x + y \leq 7$ $x > 3$

Which of these points lie in R?

A(0, 4) B(1, 2) C(1, 10) D(6, 10) E(7, 0)

A(0, 4): $y < 3$: no; $x + y = 4$: yes; $x = 0$: no

B(1, 2): $y < 5$: yes; $x + y = 3$: yes; $x = 1$: no

C(1, 10): $y < 5$: no; $x + y = 11$: no; $x = 1$: no

D(6, 10): $y < 15$: yes; $x + y = 16$: no; $x = 6$: yes

E(7, 0): $y < 17$: yes; $x + y = 7$: yes; $x = 7$: yes

In R: E

Answer: E

5 Exact trigonometric ratios from a given ratio

x is an acute angle and $\tan x = \frac{3}{11}$.

(a) Find the exact value of sin x.

(b) Find the exact value of cos x. Give each answer as a fraction or a surd with a rational denominator.

Draw a right-angled triangle with opposite side 3 and adjacent side 11

Hypotenuse = $\sqrt{3^2 + 11^2} = \sqrt{130}$

(a) $\sin x = \frac{3}{\sqrt{130}} = \frac{3\sqrt{130}}{130}$

(b) $\cos x = \frac{11}{\sqrt{130}} = \frac{11\sqrt{130}}{130}$

Answer: (a) $\frac{3\sqrt{130}}{130}$ (b) $\frac{11\sqrt{130}}{130}$

6 Ratio and algebra

In a coach, the ratio of the number of adults to the number of children is 5 : 8.

Then 20 adults are removed and 13 more children are added.

The ratio is now 2 : 5.

How many children were there at the start?

Let the numbers at the start be 5k and 8k

(5k − 20) : (8k + 13) = 2 : 5, so 5(5k − 20) = 2(8k + 13)

25k − 100 = 16k + 26

9k = 126, so k = 14

Number of children at the start: 8 × 14 = 112

Answer: 112

7 Venn diagram with algebra

The Venn diagram shows information about 15 customers.

A: ordered tea B: ordered coffee

(a) Find the value of x.

(b) Find n(A ∩ B) and n(A ∪ B)′.

(a) 7x + 1 = 15

7x = 14, so x = 2

(b) n(A ∩ B) = 2x = 4

n(A ∪ B)′ = 2 (the number outside both circles)

Answer: (a) x = 2 (b) n(A ∩ B) = 4, n(A ∪ B)′ = 2

8 Estimating the median from grouped data

The table gives information about 100 times.

Work out an estimate for the median.

Assume the values in each class are evenly spread.

Cumulative frequencies: 21, 33, 53, 83, 100

The median is the value at position 100 ÷ 2 = 50

It is in the class 20 to 30, with 33 values before it, so we need 17 more of the 20 values in this class

Median ≈ $20 + \frac{17}{20} \times 10 = 28.5$

Answer: 28.5

9 Finding a percentage rate

Kofi invests £9,500 in an account paying compound interest.
The interest rate is the same percentage each year.
After 2 years the account holds £18,620.
Work out the annual percentage interest rate.

Multiplier for 2 years = £18,620 ÷ £9,500 = 1.96
Yearly multiplier = the square root of 1.96 = 1.4
40% increase each year
Answer: 40%

10 Tangent through a given point: find the constant

A curve has equation $y = -x^2 - 5x + k$, where k is a constant.
The tangent to the curve at the point where x = 2 passes through (−4, 32).
Find the value of k.

$\frac{dy}{dx} = -2x - 5$, so at x = 2 the gradient is −9
The point of tangency is (2, −14 + k)
Gradient between this point and (−4, 32): $\frac{32 - (-14 + k)}{-4 - 2} = -9$
32 − (−14 + k) = 54
k = −8
Answer: k = −8