Day 206 **IGCSE 10-a-day: Model Solutions**

1 Expanding brackets with surds

Expand and simplify $(1 - \sqrt{3})(2 + \sqrt{3})$
Give your answer in the form $p + q\sqrt{3}$

$(1 - \sqrt{3})(2 + \sqrt{3}) = 2 + \sqrt{3} - 2\sqrt{3} - 3$
$= -1 - \sqrt{3}$
Answer: $-1 - \sqrt{3}$

2 Column vector arithmetic

$\mathbf{a} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$.
(a) Work out $\mathbf{a} + \mathbf{b}$.
(b) Work out $3\mathbf{a} - 3\mathbf{b}$.

(a) $\begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$
(b) $3\mathbf{a} = \begin{pmatrix} 12 \\ 6 \end{pmatrix}$ and $3\mathbf{b} = \begin{pmatrix} -6 \\ 0 \end{pmatrix}$
$\begin{pmatrix} 12 \\ 6 \end{pmatrix} - \begin{pmatrix} -6 \\ 0 \end{pmatrix} = \begin{pmatrix} 18 \\ 6 \end{pmatrix}$
Answer: (a) $\begin{pmatrix} \mathbf{2} \\ \mathbf{2} \end{pmatrix}$ (b) $\begin{pmatrix} \mathbf{18} \\ \mathbf{6} \end{pmatrix}$

3 Vertical distance between stationary points

The curve $y = x^3 + 3x^2 + 9$ has two stationary points, A and B.
Work out the vertical distance between A and B.

$\frac{dy}{dx} = 3x^2 + 6x = 3x(x + 2)$
$x = -2$ and $x = 0$
At $x = -2$, $y = 13$; at $x = 0$, $y = 9$
Difference $= 13 - 9 = 4$
Answer: 4

4 Two picks without replacement

A bag contains 4 red and 6 yellow cubes.
Two cubes are taken at random without replacement.
Work out the probability that both are red.

P(first red) $= \frac{4}{10}$
P(second red) $= \frac{3}{9}$
P(both red) $= \frac{4}{10} \times \frac{3}{9} = \frac{2}{15}$
Answer: $\frac{2}{15}$

5 Transformations of graphs

The point P(6, 2) lies on the graph of $y = f(x)$.
Write down the coordinates of the image of P on the graph of
(a) $y = f(x) + 2$ (b) $y = f(-x)$

(a) $y = f(x) + 2$ is a translation of 2 up: (6, 4)
(b) $y = f(-x)$ is a reflection in the y-axis, so the x coordinate changes sign: $(-6, 2)$
Answer: (a) (6, 4) (b) $(-6, 2)$

6 Equation of a normal

A curve has equation $y = 3x^3 - 3x + 2$.
Find the equation of the normal to the curve at the point where $x = -1$.

When $x = -1$, $y = 2$ and $\frac{dy}{dx} = 6$

Gradient of the normal $= -1 \div 6 = -\frac{1}{6}$

$y - 2 = -\frac{1}{6}(x + 1)$

$y = -\frac{1}{6}x + \frac{11}{6}$

Answer: $y = -\frac{1}{6}x + \frac{11}{6}$

7 HCF and LCM

$A = 2^3 \times 7^3$
$B = 2^3 \times 7$
Write down, as a product of prime factors,
(a) the HCF of A and B
(b) the LCM of A and B

HCF: take the lowest power of each prime that appears in both numbers
$= 2^3 \times 7$
LCM: take the highest power of every prime that appears
$= 2^3 \times 7^3$

Answer: (a) $2^3 \times 7$ (b) $2^3 \times 7^3$

8 Inverse proportion

T is inversely proportional to the square root of n.
When n = 4, T = 3.
Find the value of T when n = 1.

$T = \frac{k}{\sqrt{n}}$

$3 = \frac{k}{2}$, so $k = 3 \times 2 = 6$

$T = \frac{6}{1} = 6$

Answer: T = 6

9 Vector proof: collinear points

OA = **a** and OB = **b**.
OP = 4**a** + 5**b**, OQ = 7**a** + 7**b** and OR = 19**a** + 15**b**.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = 3**a** + 2**b**
QR = OR − OQ = 12**a** + 8**b**
QR = 4 PQ, so PQ and QR are parallel, and they share the point Q
So P, Q and R lie on a straight line
(b) PQ : QR = 1 : 4

Answer: (b) 1 : 4

10 Three stationary points of a quartic

$y = -x^4 + 8x^2 + 4$

Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = -4x^3 + 16x = -4x(x - 2)(x + 2)$

$\frac{dy}{dx} = 0$ gives $x = -2$, $x = 0$ or $x = 2$

Points: $(-2, 20)$, $(0, 4)$, $(2, 20)$

$\frac{d^2y}{dx^2} = -12x^2 + 16$

$x = -2$: $\frac{d^2y}{dx^2} = -32$; $x = 0$: $\frac{d^2y}{dx^2} = 16$; $x = 2$: $\frac{d^2y}{dx^2} = -32$

So $(-2, 20)$ is a maximum, $(0, 4)$ is a minimum, $(2, 20)$ is a maximum

Answer: $(-2, 20)$ maximum; $(0, 4)$ minimum; $(2, 20)$ maximum