Day 205 **IGCSE 10-a-day: Model Solutions**

1 Exact trigonometric values

Work out the exact value of $2 \tan 30° \times \cos 60°$.
Give your answer in its simplest form, with no decimals.

$\tan 30° = \frac{\sqrt{3}}{3}$ and $\cos 60° = \frac{1}{2}$

Substitute and simplify (rationalise any denominator)

$= \frac{\sqrt{3}}{3}$

Answer: $\frac{\sqrt{3}}{3}$

2 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle OAB = 31°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) OA = OB (radii), so triangle OAB is isosceles and angle OBA = 31°
Angle AOB = $180 - 2 \times 31 = 118°$
(b) The angle at the circumference is half the angle at the centre: $118 \div 2 = 59°$
Answer: (a) 118° (b) 59°

3 Same gradient on two curves

Two curves have equations $y = 3x^2 - 2x + 5$ and $y = x^2 - 10x - 5$.
Find the value of x at which the curves have the same gradient.

First curve: $\frac{dy}{dx} = 6x - 2$

Second curve: $\frac{dy}{dx} = 2x - 10$

Equal gradients: $6x - 2 = 2x - 10$
$4x = -8$, so $x = -2$
Answer: x = −2

4 Similar shapes

Two shapes are mathematically similar.
The smaller has height 27 cm and area 28 cm^2.
The larger has area $\frac{112}{9}$ cm^2.
Work out the height of the larger shape.

Ratio of areas = $\frac{112}{9} : 28 = 4 : 9$

Ratio of lengths = 2 : 3

Height = $27 \times \frac{2}{3} = 18$ cm

Answer: 18 cm

5 Cumulative frequency graphs

The cumulative frequency graph gives information about the times of 120 students to complete a puzzle.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 120 had a time greater than 65.

(a) Median at cumulative frequency 60, so median = 50
Lower quartile at 30, so LQ = 40. Upper quartile at 90, so UQ = 60
Interquartile range = $60 - 40 = 20$
(b) At 65, cumulative frequency = 101
Number greater = $120 - 101 = 19$
Answer: (a) median 50, IQR 20 (b) 19

6 Intersection and gradient

The curve $y = x^2 + 2x$ and the line $y = x + 2$ intersect at two points A and B, where A has the smaller x coordinate.
(a) Find the coordinates of A and B.
(b) Find the gradient of the curve at A and at B.

$x^2 + 2x = x + 2$ gives $x^2 + x - 2 = 0$, so $(x + 2)(x - 1) = 0$
$x = -2$ or $x = 1$, giving A $(-2, 0)$ and B $(1, 3)$
$\frac{dy}{dx} = 2x + 2$: at A it is -2, at B it is 4
Answer: (a) A $(-2, 0)$, B $(1, 3)$ (b) -2 at A, 4 at B

7 Finding a matrix from MA = B

$A = \begin{pmatrix} 0 & -1 \\ 1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & -6 \\ 3 & 9 \end{pmatrix}$.
MA = B, where M is a 2 × 2 matrix.
Find M.

MA = B so $M = BA^{-1}$
$A^{-1} = \begin{pmatrix} 4 & 1 \\ -1 & 0 \end{pmatrix}$
$M = \begin{pmatrix} -1 & -6 \\ 3 & 9 \end{pmatrix}\begin{pmatrix} 4 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 3 & 3 \end{pmatrix}$
Answer: $\mathbf{M} = \begin{pmatrix} \mathbf{2} & \mathbf{-1} \\ \mathbf{3} & \mathbf{3} \end{pmatrix}$

8 Curve touching the x-axis

The curve $y = 2x^3 + 18x^2 + 48x + k$, where k is a constant, has the x-axis as a tangent.
Find the two possible values of k.

$\frac{dy}{dx} = 6x^2 + 36x + 48 = 0$ gives $x = -4$ or $x = -2$
The x-axis is a tangent when a turning point has $y = 0$
$x = -4$: $-32 + k = 0$, so $k = 32$
$x = -2$: $-40 + k = 0$, so $k = 40$
Answer: $k = 32$ or $k = 40$

9 Equal roots and real roots

Show that the equation $x^2 + 3kx - 2 = 0$ has two distinct real solutions for all values of k.

$b^2 - 4ac = (3k)^2 - 4 \times 1 \times (-2)$
$= 9k^2 + 8$
$9k^2$ is never negative, so $9k^2 + 8$ is always at least 8, which is greater than 0
So $b^2 - 4ac > 0$ for all k, and there are always two distinct real solutions
Answer: Shown

10 Bounds of speed, density and pressure

A train travels a distance of 400 km, correct to the nearest 50.
The journey takes 4.3 hours, correct to 1 decimal place.
Work out the lower bound of the speed.
Give your answer correct to 3 significant figures. (Calculator)

$375 \leq$ distance < 425; $4.25 \leq$ time < 4.35
Lower bound of speed = smallest distance ÷ biggest time = 375 ÷ 4.35
= 86.2069... = 86.2 (3 s.f.)
Answer: 86.2 km/h